ARTICLE IN PRESS YJDEQ:9152

Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

On center singularity for compressible spherically symmetric nematic liquid crystal flows

Yun Wang [a,*], Xiangdi Huang [b]

[a] *Department of Mathematics, Soochow University, SuZhou, 215006, People's Republic of China*

[b] *NCMIS, AMSS, Chinese Academy of Sciences, Beijing, 100190, People's Republic of China*

Received 9 June 2015; revised 29 November 2017

Abstract

The paper is concerned with a simplified system, proposed by Ericksen [6] and Leslie [20], modeling the flow of nematic liquid crystals. In the first part, we give a new Serrin's continuation principle for strong solutions of general compressible liquid crystal flows. Based on new observations, we establish a localized Serrin's regularity criterion for the 3D compressible spherically symmetric flows. It is proved that the classical solution loses its regularity in finite time if and only if, either the concentration or vanishing of mass forms or the norm inflammation of gradient of orientation field occurs around the center.

MSC: 35Q35; 35B65; 76N10

Keywords: Compressible nematic liquid crystal flows; Blowup criterion; Spherically symmetric flows

1. Introduction

In this paper, we consider the motion of the compressible nematic liquid crystal flows, which is described by the following simplified Ericksen–Leslie system,

* Corresponding author.
E-mail addresses: ywang3@suda.edu.cn (Y. Wang), xdhuang@amss.ac.cn (X. Huang).

https://doi.org/10.1016/j.jde.2017.12.035

$$\begin{cases} \rho_t + \mathrm{div}(\rho U) = 0, \\ \rho U_t + \rho(U\cdot\nabla)U - \mu\Delta U - (\mu+\lambda)\nabla(\mathrm{div}U) + \nabla P = -\Delta d\cdot\nabla d, \\ d_t + (U\cdot\nabla)d = \Delta d + |\nabla d|^2 d. \end{cases} \tag{1.1}$$

Here $\rho \geq 0$ is the density of the fluid and U is the velocity field. d denotes the macroscopic average of the nematic liquid crystal orientation field, which conforms to $|d| = 1$. $P = P(\rho)$ is the pressure of the fluid, which is a function of the density. The equation of state is given by

$$P(\rho) = a\rho^\gamma, \quad a > 0,\ \gamma > 1. \tag{1.2}$$

The constants μ and λ are the shear viscosity and the bulk viscosity coefficients of the fluid respectively, and they satisfy the following physical conditions,

$$\mu > 0, \quad 3\lambda + 2\mu \geq 0. \tag{1.3}$$

The three equations in (1.1) are the equations for conservation of mass, linear momentum and angular momentum respectively.

The domain Ω is a bounded ball with radius R, namely,

$$\Omega = B_R = \{x \in \mathbb{R}^3;\ |x| \leq R < \infty\}.$$

We study an initial boundary value problem for (1.1) with the initial condition

$$(\rho, U, d)(0, x) = (\rho_0, U_0, d_0)(x), \quad x \in \Omega, \tag{1.4}$$

and the boundary condition

$$U(t, x) = 0, \quad \frac{\partial d}{\partial \vec{n}}(t, x) = 0, \quad t \geq 0,\ x \in \partial\Omega. \tag{1.5}$$

And we are looking for the smooth spherically symmetric solution (ρ, U, d) of the problem (1.1)–(1.5) which enjoys the form

$$\rho(t, x) = \rho(t, |x|), \quad U(t, x) = u(t, |x|)\frac{x}{|x|}, \quad d(t, x) = d(t, |x|). \tag{1.6}$$

Then, for the initial data to be consistent with the form (1.7), we assume the initial data (ρ_0, U_0, d_0) also takes the form

$$\rho_0(x) = \rho_0(|x|), \quad U_0(x) = u_0(|x|)\frac{x}{|x|}, \quad d_0(x) = d_0(|x|). \tag{1.7}$$

In this paper, we further assume the initial density is uniformly positive, that is,

$$\rho_0(x) = \rho_0(|x|) \geq \underline{\rho} > 0, \quad x \in \Omega, \tag{1.8}$$

Please cite this article in press as: Y. Wang, X. Huang, On center singularity for compressible spherically symmetric nematic liquid crystal flows, J. Differential Equations (2018), https://doi.org/10.1016/j.jde.2017.12.035