



# On hyperbolicity and Gevrey well-posedness. Part two: Scalar or degenerate transitions

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## Abstract

For first-order quasi-linear systems of partial differential equations, we formulate an assumption of a transition from initial hyperbolicity to ellipticity. This assumption bears on the principal symbol of the first-order operator. Under such an assumption, we prove a strong Hadamard instability for the associated Cauchy problem, namely an instantaneous defect of Hölder continuity of the flow from  $G^\sigma$  to  $L^2$ , with  $0 < \sigma < \sigma_0$ , the limiting Gevrey index  $\sigma_0$  depending on the nature of the transition. We restrict here to scalar transitions, and non-scalar transitions in which the boundary of the hyperbolic zone satisfies a flatness condition. As in our previous work for initially elliptic Cauchy problems [B. Morisse, *On hyperbolicity and Gevrey well-posedness. Part one: the elliptic case*, arXiv:1611.07225], the instability follows from a long-time Cauchy–Kovalevskaya construction for highly oscillating solutions. This extends recent work of N. Lerner, T. Nguyen, and B. Texier [*The onset of instability in first-order systems*, to appear in J. Eur. Math. Soc.].

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## 1. Introduction

We consider the following Cauchy problem, for first-order quasi-linear systems of partial differential equations:

$$\partial_t u = \sum_{j=1}^d A_j(t, x, u) \partial_{x_j} u + f(t, x, u), \quad u(0, x) = h(x). \quad (1.1)$$

The system is of size  $N$ , that is  $u(t, x)$  and  $f(t, x, u)$  are in  $\mathbb{R}^N$  and the  $A_j(t, x, u) \in \mathbb{R}^{N \times N}$ . The time  $t$  is nonnegative, and  $x$  is in  $\mathbb{R}^d$ . We assume throughout the paper that the  $A_j$  and  $f$  are analytic in a neighborhood of some point  $(0, x_0, u_0) \in \mathbb{R}_t \times \mathbb{R}_x^d \times \mathbb{R}_u^N$ .

Under assumptions of weak defects of hyperbolicity for the first-order operator, we prove ill-posedness of (1.1) in Gevrey spaces. Weak defect of hyperbolicity is here understood as a transition from hyperbolicity of the principal symbol at initial time, to ellipticity of the principal symbol for later times. Our results extend Métivier's ill-posedness theorem in Sobolev spaces for initially elliptic operators [10], our own ill-posedness result in Gevrey spaces for initially elliptic operators [11], Lerner, Nguyen and Texier's theorem on systems transitioning from hyperbolicity to ellipticity [6], and echo Lu's construction of WKB profiles [8] which are destabilized by terms not present in the initial data.

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