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On hyperbolicity and Gevrey well-posedness. Part two: Scalar or degenerate transitions

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Abstract

For first-order quasi-linear systems of partial differential equations, we formulate an assumption of a transition from initial hyperbolicity to ellipticity. This assumption bears on the principal symbol of the first-order operator. Under such an assumption, we prove a strong Hadamard instability for the associated Cauchy problem, namely an instantaneous defect of Hölder continuity of the flow from G^σ to L^2, with $0 < \sigma < \sigma_0$, the limiting Gevrey index σ_0 depending on the nature of the transition. We restrict here to scalar transitions, and non-scalar transitions in which the boundary of the hyperbolic zone satisfies a flatness condition. As in our previous work for initially elliptic Cauchy problems [B. Morisse, *On hyperbolicity and Gevrey well-posedness. Part one: the elliptic case*, arXiv:1611.07225], the instability follows from a long-time Cauchy–Kovalevskaya construction for highly oscillating solutions. This extends recent work of N. Lerner, T. Nguyen, and B. Texier [*The onset of instability in first-order systems*, to appear in J. Eur. Math. Soc.].

Contents

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1. Introduction

We consider the following Cauchy problem, for first-order quasi-linear systems of partial differential equations:

$$\partial_t u = \sum_{j=1}^{d} A_j(t,x,u)\partial_{x_j} u + f(t,x,u)\,, \qquad u(0,x) = h(x). \tag{1.1}$$

The system is of size N, that is $u(t,x)$ and $f(t,x,u)$ are in $\mathbb{R}^N$ and the $A_j(t,x,u) \in \mathbb{R}^{N\times N}$. The time t is nonnegative, and x is in $\mathbb{R}^d$. We assume throughout the paper that the A_j and f are analytic in a neighborhood of some point $(0,x_0,u_0) \in \mathbb{R}_t \times \mathbb{R}^d_x \times \mathbb{R}^N_u$.

Under assumptions of weak defects of hyperbolicity for the first-order operator, we prove ill-posedness of (1.1) in Gevrey spaces. Weak defect of hyperbolicity is here understood as a transition from hyperbolicity of the principal symbol at initial time, to ellipticity of the principal symbol for later times. Our results extend Métivier's ill-posedness theorem in Sobolev spaces for initially elliptic operators [10], our own ill-posedness result in Gevrey spaces for initially elliptic operators [11], Lerner, Nguyen and Texier's theorem on systems transitioning from hyperbolicity to ellipticity [6], and echo Lu's construction of WKB profiles [8] which are destabilized by terms not present in the initial data.