



# Iterated Hamiltonian type systems and applications

Dan Tiba

*Academy of Romanian Scientists and Institute of Mathematics, Romanian Academy, P.O. BOX 1-764,  
014700 Bucharest, Romania*

Received 20 November 2016; revised 19 September 2017

---

## Abstract

We discuss, in arbitrary dimension, certain Hamiltonian type systems and prove existence, uniqueness and regularity properties, under the independence condition. We also investigate the critical case, define a class of generalized solutions and prove existence and basic properties. Relevant examples and counterexamples are also indicated. The applications concern representations of implicitly defined manifolds and their perturbations, motivated by differential systems involving unknown geometries.

© 2018 Published by Elsevier Inc.

MSC: 34A34; 26B10; 34A12; 49K21; 49M37

Keywords: Uniqueness; Critical case; Generalized solutions; Local parametrizations

---

## 1. Introduction

In the bounded domain  $\Omega$  in the Euclidean space  $R^d$ ,  $d \in N$ , we consider a family of  $C^1$  mappings  $F_1, F_2, \dots, F_l$ ,  $l \leq d - 1$  and we associate to them certain iterated systems of Hamiltonian type equations such that  $F_1, F_2, \dots, F_l$  are constant on the corresponding trajectories. In the classical case of Hamiltonian systems this corresponds to the conservation of the energy. In dimension two and three, such systems were discussed in the recent papers [24], [16], including the critical case. Several relevant numerical examples are also indicated in these works. In fact, in dimension two, the classical Hamiltonian (non critical) case appears for instance in Thorpe, [22], p. 63, in a similar context.

---

*E-mail address:* [dan.tiba@imar.ro](mailto:dan.tiba@imar.ro).

<https://doi.org/10.1016/j.jde.2018.01.003>

0022-0396/© 2018 Published by Elsevier Inc.

We discuss here the general case in arbitrary dimension and prove existence, regularity and uniqueness. The systems of ordinary differential equations that we use are new and are derived from a first order partial differential system of equations. The applications, in the second part of Section 2, are related to representations of implicitly defined manifolds and their perturbations, with reference to fixed domain methods in shape optimization problems, [15], [25], [14].

In Section 3, we apply the above results in order to solve the critical case as well, under the assumption  $F_j \in C^1(\Omega)$ ,  $j = \overline{1, l}$ , and in the absence of independence conditions. We introduce the notion of (local) generalized solution of the system (2.6) via perturbations of the initial conditions in the Hamiltonian equations, prove its existence and basic properties. We also indicate some relevant examples. The generalized solution obtained by our method is novel, covers all possible cases and is a generalization of the classical notion of local solution. Singular situations were discussed by different methods in [2], [4], [12] and a comprehensive account can be found in [9], Ch. 5.4, where it is specified that a complete solution of the critical case is not known.

There is a recent interest in this direction due to important questions in the well known level set method for evolving surfaces, see [10] and its references, or in shape optimization, [25], [5], [23], in free boundary problems [7], [13]. Integrating over implicitly defined manifolds can be handled by the implicit parametrization result that we prove in Theorem 4.

## 2. Iterated Hamiltonian type systems

In this section, we impose the classical independence assumption on the family of  $C^1$  mappings  $F_1, F_2, \dots, F_l$ , in some point  $x^0 \in \Omega \subset \mathbb{R}^d$ ,  $l \leq d - 1$ . To fix ideas, we assume

$$\frac{D(F_1, F_2, \dots, F_l)}{D(x_1, x_2, \dots, x_l)} \neq 0 \quad \text{in } x^0 = (x_1^0, x_2^0, \dots, x_d^0). \quad (2.1)$$

The hypothesis (2.1) will be dropped in the next section.

Clearly, condition (2.1) remains valid on a neighborhood  $V \in \mathcal{V}(x^0)$ ,  $V \subset \Omega$ , under the  $C^1(\Omega)$  assumption on  $F_j(\cdot)$ ,  $j = \overline{1, l}$  and we denote by  $A(x)$ ,  $x \in V$ , the corresponding nonsingular  $l \times l$  matrix from (2.1).

We introduce on  $V$  the undetermined linear algebraic system with unknowns  $v(x) \in \mathbb{R}^d$ ,  $x \in V$ :

$$v(x) \cdot \nabla F_j(x) = 0, \quad j = \overline{1, l}. \quad (2.2)$$

We shall use  $d - l$  solutions of (2.2) obtained by fixing successively the last  $d - l$  components of the vector  $v(x) \in \mathbb{R}^d$  to be the rows of the identity matrix in  $\mathbb{R}^{d-l}$  multiplied by  $\Delta(x) = \det A(x)$ . Then, the first  $l$  components are uniquely determined, by inverting  $A(x)$ , due to (2.1).

In this way, the obtained  $d - l$  solutions of (2.2), denoted by  $v_1(x), \dots, v_{d-l}(x) \in \mathbb{R}^d$ , are linear independent, for any  $x \in V$ .

Moreover, these vector fields are continuous in  $V$  as  $\nabla F_j(\cdot)$  are continuous in  $V$  and the Cramer's rule ensures the continuity with respect to the coefficients of the solution for linear systems. Other useful choices of solutions for (2.2), are possible (see Theorem 5).

We introduce now  $d - l$  nonlinear systems of first order partial differential equations associated to the vector fields  $(v_j(x))_{j=\overline{1, d-l}}$ ,  $x \in V \subset \Omega$ . Furthermore, we denote the sequence of independent variables by  $t_1, t_2, \dots, t_{d-l}$ .

Download English Version:

<https://daneshyari.com/en/article/8898865>

Download Persian Version:

<https://daneshyari.com/article/8898865>

[Daneshyari.com](https://daneshyari.com)