ARTICLE IN PRESS YJDEQ:9126

Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Asymptotics of viscoelastic materials with nonlinear density and memory effects ☆

M. Conti [a], T.F. Ma [b,*], E.M. Marchini [a], P.N. Seminario Huertas [b]

[a] *Politecnico di Milano, Dipartimento di Matematica, Via Bonardi 9, 20133 Milano, Italy*
[b] *Departamento de Matemática, Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo, 13566-590 São Carlos, SP, Brazil*

Received 27 January 2017; revised 7 August 2017

Abstract

This paper is concerned with the nonlinear viscoelastic equation

$$|\partial_t u|^\rho \partial_{tt} u - \Delta \partial_{tt} u - \Delta u + \int_0^\infty \mu(s)\Delta u(t-s)\,\mathrm{d}s + f(u) = h,$$

suitable to modeling extensional vibrations of thin rods with nonlinear material density $\varrho(\partial_t u) = |\partial_t u|^\rho$, and presence of memory effects. This class of equations was studied by many authors, but well-posedness in the whole admissible range $\rho \in [0, 4]$ and for f growing up to the critical exponent were established only recently. The existence of global attractors was proved in presence of an additional viscous or frictional damping. In the present work we show that the sole weak dissipation given by the memory term is enough to ensure existence and optimal regularity of the global attractor $\mathcal{A}_\rho$ for $\rho < 4$ and critical nonlinearity f.

MSC: 37B55; 35L70; 35B41

☆ The first and the third authors are partially supported by the research project GNAMPA-INdAM 2015 “Proprietà asintotiche di sistemi differenziali con memoria degenere”, the second author by CNPq grant 310041/2015-5, and the fourth by CAPES/PROEX grant 8477445/D.

* Corresponding author.

E-mail addresses: monica.conti@polimi.it (M. Conti), matofu@icmc.usp.br (T.F. Ma), elsa.marchini@polimi.it (E.M. Marchini), pseminar@icmc.usp.br (P.N. Seminario Huertas).

https://doi.org/10.1016/j.jde.2017.12.010
0022-0396/

Please cite this article in press as: M. Conti et al., Asymptotics of viscoelastic materials with nonlinear density and memory effects, J. Differential Equations (2017), https://doi.org/10.1016/j.jde.2017.12.010

Keywords: Viscoelastic equation; Memory; Nonlinear density; Global attractors

1. Introduction

In the recent literature the nonlinear equation

$$|\partial_t u|^\rho \partial_{tt} u - \Delta\partial_{tt} u - \gamma\Delta\partial_t u - \Delta u + \int_0^t \mu(t-s)\Delta u(s)\,\mathrm{d}s + \mathcal{F} = 0 \text{ in } \Omega\times\mathbb{R}^+, \tag{1.1}$$

has been considered by many authors. The problem is usually set in a bounded domain Ω of $\mathbb{R}^N$ with

$$0 < \rho \leq \frac{2}{N-2} \text{ if } N \geq 3 \text{ and } \rho > 0 \text{ if } N = 1, 2, \tag{1.2}$$

where $\gamma \geq 0$ accounts for the structural mechanical damping, μ is a nonnegative nonincreasing function describing memory effects in the material, and $\mathcal{F}$ represents forcing terms. Its motivation comes from the prototype equation

$$\varrho\partial_{tt} u - \Delta\partial_{tt} u - \Delta u = 0, \tag{1.3}$$

which models several applications, among them, extensional vibrations of thin roads [19, Chap. 20] and ion-sound waves [3, Sec. 6]. However, as observed in [6], in certain cases the material density may depend on small variations of the velocity, that is, $\varrho = \varrho(\partial_t u)$. Then a natural assumption would be

$$\varrho(\partial_t u) = 1 + \epsilon\,|\partial_t u|^\rho, \quad \rho > 0,$$

where $\epsilon \in \mathbb{R}$ is a small parameter. By simplicity, without loss of mathematical complexity, we shall assume $\varrho(\partial_t u) = |\partial_t u|^\rho$, $\rho > 0$. From this understanding, we obtain equation (1.1) by adding memory effects and damping to the model (1.3).

Equation (1.1) was firstly studied in [5] with $\mathcal{F} = 0$, where the existence of global solutions is proved; besides, in the case $\gamma > 0$, which corresponds to the strongly damped problem, the decay of the energy as $t \to \infty$ is established. Later, many authors have contributed to this problem in several directions, cf. [6,15–18,20–23,25,32], always assuming condition (1.2).

On the other hand, the uniqueness of solutions to this class of problem was not considered until the recent paper [1], in the more general context with infinite delay

$$|\partial_t u|^\rho \partial_{tt} u - \Delta\partial_{tt} u - \gamma\Delta\partial_t u - \Delta u + \int_0^\infty \mu(s)\Delta u(t-s)\,\mathrm{d}s + f(u) = h, \tag{1.4}$$

complemented by the Dirichlet boundary condition

$$u(\boldsymbol{x}, t)_{|\boldsymbol{x}\in\partial\Omega} = 0. \tag{1.5}$$