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Y spaces and global smooth solution of fractional Navier–Stokes equations with initial value in the critical oscillation spaces ☆

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Abstract

For fractional Navier–Stokes equations and critical initial spaces X, one used to establish the well-posedness in the solution space which is contained in $C(\mathbb{R}_+, X)$. In this paper, for heat flow, we apply parameter Meyer wavelets to introduce Y spaces $Y^{m,\beta}$ where $Y^{m,\beta}$ is not contained in $C(\mathbb{R}_+, \dot{B}^{1-2\beta,\infty}_{\infty})$. Consequently, for $\frac{1}{2} < \beta < 1$, we establish the global well-posedness of fractional Navier–Stokes equations with small initial data in all the critical oscillation spaces. The critical oscillation spaces may be any Besov–Morrey spaces $\big(\dot{B}^{\gamma_1,\gamma_2}_{p,q}(\mathbb{R}^n)\big)^n$ or any Triebel–Lizorkin–Morrey spaces $\big(\dot{F}^{\gamma_1,\gamma_2}_{p,q}(\mathbb{R}^n)\big)^n$ where $1 \leq p, q \leq \infty, 0 \leq \gamma_2 \leq \frac{n}{p}, \gamma_1 - \gamma_2 = 1 - 2\beta$. These critical spaces include many known spaces. For example, Besov spaces, Sobolev spaces, Bloch spaces, Q-spaces, Morrey spaces and Triebel–Lizorkin spaces etc.

MSC: primary 35Q30, 76D03, 42B35, 46E30

Keywords: Fractional Navier–Stokes equations; Parameter Meyer wavelets; Besov–Morrey spaces; Triebel–Lizorkin–Morrey spaces; Well-posedness; Y spaces

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Contents

1. Introduction

For $\beta > 1/2$, the Cauchy problem of the so-called Fractional Navier–Stokes (FNS) equations on the half-space $\mathbb{R}^{1+n}_+ = (0, \infty) \times \mathbb{R}^n$, $n \geq 2$, is defined as:

$$\begin{cases} \frac{\partial u}{\partial t} + (-\Delta)^\beta u + u \cdot \nabla u - \nabla p = 0, & \text{in } \mathbb{R}^{1+n}_+; \\ \nabla \cdot u = 0, & \text{in } \mathbb{R}^{1+n}_+; \\ u|_{t=0} = a, & \text{in } \mathbb{R}^n, \end{cases} \tag{1.1}$$

where $(-\Delta)^\beta$ represents the β-order Laplace operator defined by the Fourier transform in the space variable:

$$\widehat{(-\Delta)^\beta} u(t, \xi) = |\xi|^{2\beta} \hat{u}(t, \xi).$$

Upon letting R_j, $j = 1, 2, \cdots n$ be the Riesz transforms, writing

$$\begin{cases} \mathbb{P} = \{\delta_{l,l'} + R_l R_{l'}\}, l, l' = 1, \cdots, n; \\ \mathbb{P}\nabla(u \otimes u) = \sum\limits_l \frac{\partial}{\partial x_l}(u_l u) - \sum\limits_l \sum\limits_{l'} R_l R_{l'} \nabla(u_l u_{l'}); \\ e^{-t\widehat{(-\Delta)^\beta}} f(\xi) = e^{-t|\xi|^{2\beta}} \hat{f}(\xi), \end{cases} \tag{1.2}$$

and using $\nabla \cdot u = 0$, we can see that solutions of the above Cauchy problem are then obtained via the integral equation:

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