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J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

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Nonlinear second order evolution inclusions with noncoercive viscosity term

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Received 27 March 2017; revised 1 December 2017

Abstract

In this paper we deal with a second order nonlinear evolution inclusion, with a nonmonotone, noncoercive viscosity term. Using a parabolic regularization (approximation) of the problem and *a priori* bounds that permit passing to the limit, we prove that the problem has a solution.

MSC: primary 35L90; secondary 35R70, 47H04, 47H05

Keywords: Evolution triple; Compact embedding; Parabolic regularization; Noncoercive viscosity term; A priori bounds

1. Introduction

Let $T = [0, b]$ and let (X, H, X^*) be an evolution triple of spaces, with the embedding of X into H being compact (see Section 2 for definitions).

In this paper, we study the following nonlinear evolution inclusion:

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https://doi.org/10.1016/j.jde.2017.12.022
0022-0396/

Please cite this article in press as: N.S. Papageorgiou et al., Nonlinear second order evolution inclusions with noncoercive viscosity term, J. Differential Equations (2017), https://doi.org/10.1016/j.jde.2017.12.022

$$\left\{\begin{array}{l} u''(t) + A(t, u'(t)) + Bu(t) \in F(t, u(t), u'(t)) \text{ for almost all } t \in T, \\ u(0) = u_0,\ u'(0) = u_1. \end{array}\right\} \tag{1}$$

In the past, such multi-valued problems were studied by Gasinski [3], Gasinski and Smolka [6,7], Migórski et al. [11–14], Ochal [15], Papageorgiou, Rădulescu and Repovš [16,17], Papageorgiou and Yannakakis [18,19]. The works of Gasinski [3], Gasinski and Smolka [6,7] and Ochal [15], all deal with hemivariational inequalities, that is, $F(t, x, y) = \partial J(x)$ with $J(\cdot)$ being a locally Lipschitz functional and $\partial J(\cdot)$ denoting the Clarke subdifferential of $J(\cdot)$. In Papageorgiou and Yannakakis [18,19], the multivalued term $F(t, x, y)$ is general (not necessarily of the subdifferential type) and depends also on the time derivative of the unknown function $u(\cdot)$. With the exception of Gasinski and Smolka [7], in all the other works the viscosity term $A(t, \cdot)$ is assumed to be coercive or zero. In the work of Gasinski and Smolka [7], the viscosity term is autonomous (that is, time independent) and $A : X \to X^*$ is linear and bounded.

In this work, the viscosity term $A : T \times X \to X^*$ is time dependent, noncoercive, nonlinear and nonmonotone in $x \in X$. In this way, we extend and improve the result of Gasinski and Smolka [7]. Our approach uses a kind of parabolic regularization of the inclusion, analogous to the one used by Lions [10, p. 346] in the context of semilinear hyperbolic equations.

2. Mathematical background and hypotheses

Let V, Y be Banach spaces and assume that V is embedded continuously and densely into Y (denoted by $V \hookrightarrow Y$). Then we have the following properties:

(i) Y^* is embedded continuously into V^*;

(ii) if V is reflexive, then $Y^* \hookrightarrow V^*$.

The following notion is a useful tool in the theory of evolution equations.

Definition 1. By an "evolution triple" (or "Gelfand triple") we understand a triple of spaces (X, H, X^*) such that

(a) X is a separable reflexive Banach space and X^* is its topological dual;
(b) H is a separable Hilbert space identified with its dual H^*, that is, $H = H^*$ (pivot space);
(c) $X \hookrightarrow H$.

Then from the initial remarks we have

$$X \hookrightarrow H = H^* \hookrightarrow X^*.$$

In what follows, we denote by $||\cdot||$ the norm of X, by $|\cdot|$ the norm of H and by $||\cdot||_*$ the norm of X^*. Evidently we can find $\hat{c}_1, \hat{c}_2 > 0$ such that

$$|\cdot| \leqslant \hat{c}_1 ||\cdot|| \text{ and } ||\cdot||_* \leqslant \hat{c}_2 |\cdot|.$$

By $(\cdot, \cdot)$ we denote the inner product of H and by $\langle\cdot, \cdot\rangle$ the duality brackets for the pair (X^*, X). We have

$$\langle\cdot, \cdot\rangle|_{H\times X} = (\cdot, \cdot). \tag{2}$$

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