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Resolvent estimates in homogenisation of periodic problems of fractional elasticity

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Abstract

We provide operator-norm convergence estimates for solutions to a time-dependent equation of fractional elasticity in one spatial dimension, with rapidly oscillating coefficients that represent the material properties of a viscoelastic composite medium. Assuming periodicity in the coefficients, we prove operator-norm convergence estimates for an operator fibre decomposition obtained by applying to the original fractional elasticity problem the Fourier–Laplace transform in time and Gelfand transform in space. We obtain estimates on each fibre that are uniform in the quasimomentum of the decomposition and in the period of oscillations of the coefficients as well as quadratic with respect to the spectral variable. On the basis of these uniform estimates we derive operator-norm-type convergence estimates for the original fractional elasticity problem, for a class of sufficiently smooth densities of applied forces.

MSC: 35B27; 74Q10; 34K08; 34K37; 74D1

Keywords: Fractional elasticity; Homogenisation; Gelfand transform; Operator-norm convergence; Resolvent estimates

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1. Introduction

One notable direction in the recent mathematical literature on the derivation of the overall behaviour of composites, in the context of linearised elasticity, elastodynamics, electrodynamics, is the asymptotic analysis, as the ratio ε of the microstructure size to the macroscopic size of the material sample goes to zero, of the resolvents (or "solution operators") of the (conservative) operators, elliptic in space or hyperbolic in space-time, that describe the response of the composite to exterior forces. Whenever convergence with respect to the operator norm is proved, one can often infer a host of properties about the underlying time dependent problem, in particular, the behaviour of the spectrum (*i.e.* the response to time-harmonic waves of certain frequencies and wave packets) and the convergence of the corresponding spectral projectors and operator semigroups (a version of the Trotter–Kato theorem). In the periodic setting, the advance in this kind of questions has been possible thanks to the Floquet–Bloch decomposition of the original operators into direct fibre integrals with respect to the "quasimomentum" θ and the development of various tools combining operator-theoretic considerations and the error estimates, in the spirit of classical asymptotic analysis, that are uniform in θ. The revised notion of homogenisation as the asymptotic procedure of replacing the original resolvent family by operators where different spatial scales are separated, in the sense of being described by a system of coupled field equations, can be viewed as a rigorous generalised procedure of classifying composites according to their overall response. For example, periodic composites whose component materials have highly contrasting properties (*e.g.* in the form of "soft" inclusions embedded in a "stiff" matrix) can be seen, using this kind of approach, to exhibit physical properties recognisable as those of so-called "metamaterials", *e.g.* media with negative refractive index, artificial magnetism *etc*. The asymptotically equivalent operator family found in this procedure can be viewed as an alternative model for the same composite, equivalent to it in the sense of respecting all of its qualitative and quantitative properties.

In the present work we address, for the first time, the operator-norm homogenisation-type estimates for a family of non-conservative time-dependent problems, where the energy dissipation takes place internally by friction-like forces into heat. We consider the linearised problem of one-dimensional elasticity, modified by an operator of fractional time differentiation, see [14, Section 4.2]. For the unknown scalar valued-functions u and σ, which represent the elastic displacement and stress at the point x of the medium at time t, we consider the problem

$$\begin{cases} \partial_t^2 u - \partial_x \sigma = f, \\ \sigma = (C + \partial_t^\alpha D)\partial_x u. \end{cases}$$

Here C, D are non-negative functions depending on the spatial variable $x \in \mathbb{R}$ only, which can be viewed as viscoelastic constitutive parameters of the medium, $\alpha \in (0, 1]$, and f is a given source term describing the density of forces applied to the medium. The operator ∂_t^α is the fractional time derivative in a sense to be described in the next section. If the support of f with respect to the temporal variable is bounded below, then ∂_t^α coincides with the Riemann–Liouville derivative (see *e.g.* [12,16]). We refer to [2] for a justification of the model to describe viscoelastic behaviour from an engineering perspective.

The well-posedness of the above dynamic problem has been addressed in [14], and in [25] a corresponding homogenisation problem has been considered, where convergence of the corresponding solution operators is established in a certain weak topology for operators in L^2-spaces

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