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Stability of contact discontinuities to 1-D piston problem for the compressible Euler equations

Min Ding

Department of Mathematics, School of Science, Wuhan University of Technology, Wuhan 430070, PR China

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Abstract

We consider 1-D piston problem for the compressible Euler equations when the piston is static relatively to the gas in the tube. By a modified wave front tracking method, we prove that a contact discontinuity is structurally stable under the assumptions that the total variation of the initial data and the perturbation of the piston velocity are both sufficiently small. Meanwhile, we study the asymptotic behavior of the solutions by the generalized characteristic method and approximate conservation law theory as $t \to +\infty$.

Keywords: Piston problem; Compressible Euler equations; Contact discontinuities; Wave front tracking scheme; Interaction of waves; Asymptotic behavior

1. Introduction

The piston problem is a special initial–boundary value problem in fluid dynamics which can be described as follows (see [8]). In a thin long tube that is closed at one end by a piston and open at the other end, any motion of the piston causes the corresponding motion of the gas in the tube. More precisely, if the piston is pushed forward relatively to the gas, a shock wave occurs and moves forward faster than the piston. If the piston is pulled backward relatively to the gas, a rarefaction wave appears. If the piston moves static relatively to the gas in the tube, then a contact discontinuity is generated.

E-mail address: minding@whut.edu.cn.

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In this paper, we are concerned with the 1-D piston problem when it is static relatively to the gas from the mathematical point of view. Our motivation is to establish the structural stability of the strong contact discontinuity of this problem for compressible complete Euler equations under some small perturbations of the initial data and piston velocity.

The compressible one-dimensional Euler flows consisting of the conservation laws of mass, momentum and energy can be read as:

$$\begin{cases} \partial_t \rho + \partial_x (\rho u) = 0, \\ \partial_t (\rho u) + \partial_x \left(\rho u^2 + p\right) = 0, \\ \partial_t \left(\rho(\frac{1}{2}u^2 + e)\right) + \partial_x \left(u(\frac{1}{2}\rho u^2 + \rho e + p)\right) = 0, \end{cases} \tag{1.1}$$

where ρ, p and u represent the density, the pressure and the speed of the fluid, respectively, and e is the internal energy.

For the polytropic gas, the constitutive relations are given by

$$p = \kappa \rho^\gamma \exp(S/c_v), \qquad e = \frac{p}{(\gamma - 1)\rho},$$

where S stands for the entropy, and k, c_v, $\gamma > 1$ are positive constants.

On this occasion, system (1.1) can be written in the general form of conservation law:

$$\partial_t W(U) + \partial_x H(U) = 0, \quad U = (\rho, u, p)^\top, \tag{1.2}$$

where

$$W(U) = \left(\rho,\ \rho u,\ \frac{1}{2}\rho u^2 + \frac{p}{\gamma - 1}\right)^\top,$$
$$H(U) = \left(\rho u,\ \rho u^2 + p,\ \frac{1}{2}\rho u^3 + \frac{\gamma p u}{\gamma - 1}\right)^\top.$$

By solving the polynomial $\det(\lambda \nabla_U W(U) - \nabla_U H(U)) = 0$, the eigenvalues of (1.2) are given by

$$\lambda_1 = u - c, \quad \lambda_2 = u, \quad \lambda_3 = u + c,$$

where $c = \sqrt{\frac{\gamma p}{\rho}}$ is the local sonic speed and the corresponding right eigenvectors are

$$r_1 = -\frac{2}{(\gamma+1)c}(\rho, -c, \gamma p)^\top, \quad r_2 = (1, 0, 0)^\top, \ r_3 = \frac{2}{(\gamma+1)c}(\rho, c, \gamma p)^\top,$$

where r_j is normalized by

$$\nabla \lambda_j \cdot r_j = 1, \quad j = 1, 3.$$

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