ARTICLE IN PRESS YJDEQ:9111

Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Stability results for stochastic delayed recurrent neural networks with discrete and distributed delays ☆

Guiling Chen [a], Dingshi Li [a], Lin Shi [a,*], Onno van Gaans [b], Sjoerd Verduyn Lunel [c]

[a] *School of Mathematics, Southwest Jiaotong University, Chengdu 610031, PR China*
[b] *Mathematical Institute, Leiden University, P.O. Box 9512, 2300 RA, Leiden, The Netherlands*
[c] *Mathematical Institute, Utrecht University, P.O. Box 80010, 3508 TA, Utrecht, The Netherlands*

Received 13 April 2017; revised 3 November 2017

Abstract

We present new conditions for asymptotic stability and exponential stability of a class of stochastic recurrent neural networks with discrete and distributed time varying delays. Our approach is based on the method using fixed point theory, which do not resort to any Liapunov function or Liapunov functional. Our results neither require the boundedness, monotonicity and differentiability of the activation functions nor differentiability of the time varying delays. In particular, a class of neural networks without stochastic perturbations is also considered. Examples are given to illustrate our main results.

Keywords: Fixed point theory; Asymptotic stability; Exponential stability; Stochastic recurrent neural networks; Variable delays; Doob's inequality

☆ The work is supported by the Fundamental Research Funds for the Central Universities (Grant No. A0920502051722-54 and A0920502051722-63), the Scientific Research Foundation for the introduction of talent (Grant No. A0920502051614-26), and the National Natural Science Foundation of China (Grant No. 11271270, 11701475, 11601446 and 11201320).

* Corresponding author.
E-mail addresses: guiling@home.swjtu.edu.cn (G. Chen), lidingshi2006@163.com (D. Li), shilin@swjtu.edu.cn (L. Shi), vangaans@math.leidenuniv.nl (O. van Gaans), S.M.VerduynLunel@uu.nl (S. Verduyn Lunel).

https://doi.org/10.1016/j.jde.2017.11.032
0022-0396/

Please cite this article in press as: G. Chen et al., Stability results for stochastic delayed recurrent neural networks with discrete and distributed delays, J. Differential Equations (2017), https://doi.org/10.1016/j.jde.2017.11.032

1. Introduction and main results

Neural networks have received an increasing interest in various areas [3,5]. The stability of neural networks is critical for signal processing, especially in image processing and solving some classes of optimization problems [4,13,32,33]. For the stochastic effects to the dynamical behaviors of neural networks, Liao and Mao [11,12] initiated the study of stability and instability of stochastic neural networks.

Due to the finite switching speed of neurons and amplifiers, time delays which may lead to instability and bad performance in neural processing and signal transmission are commonly encountered in both biological and artificial neural networks. In addition, neural networks usually have a spatial extent due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths [27]. Thus there will be a distribution of conduction velocities along these pathways and a distribution of propagation delays [34]. In these circumstances the signal propagation is not instantaneous and may not be suitably modeled with discrete delays. Therefore, a more appropriate way which incorporates continuously distributed delays in neural network models has been used. Further, due to random fluctuations and probabilistic causes in the network, noises do exist in a neural network. Thus, it is necessary and rewarding to study stability properties of stochastic delayed neural networks.

Liapunov's direct method has long been viewed the main classical method of studying stability problems in many areas of differential equations. The success of Liapunov's direct method depends on finding a suitable Liapunov function or Liapunov functional. However, it may be difficult to look for a good Liapunov functional for some classes of stochastic delay differential equations. Therefore, an alternative may be explored to overcome such difficulties.

It was proposed by Burton [2] and his co-workers to use a fixed point method to study the stability problem for deterministic systems. Luo [16] and Appleby [1] have applied this method to deal with the stability problems for stochastic delay differential equations, and afterwards, a great number of classes of stochastic delay differential equations are investigated by using fixed point methods, see, for example, [3,17,18,21,22]. It turns out that the fixed point method is a powerful technique in dealing with stability problems for deterministic and stochastic differential equations with delays. Moreover, it has an advantage that it can yield the existence, uniqueness and stability criteria of the considered system in one step.

In this paper, we consider a general class of stochastic neural networks with discrete and distributed varying delays which is described by

$$dx_i(t) = \left[-c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(x_j(t-\tau(t))) + \sum_{j=1}^{n} l_{ij} \int_{t-r(t)}^{t} h_j(x_j(s))\,ds\right] dt$$
$$+ \sum_{j=1}^{n} \sigma_{ij}(t, x_j(t), x_j(t-\tau(t)))\,dw_j(t), \tag{1}$$

or

$$dx(t) = \left[-Cx(t) + Af(x(t)) + Bg(x(t-\tau(t))) + W \int_{t-r(t)}^{t} h(x(s))\,ds\right] dt$$
$$+\sigma(t, x(t), x(t-\tau(t)))\,dw(t),$$