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Standing waves in a counter-rotating vortex filament pair

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Abstract

The distance among two counter-rotating vortex filaments satisfies a beam-type of equation according to the model derived in [15]. This equation has an explicit solution where two straight filaments travel with constant speed at a constant distance. The boundary condition of the filaments is 2π-periodic. Using the distance of the filaments as bifurcating parameter, an infinite number of branches of periodic standing waves bifurcate from this initial configuration with constant rational frequency along each branch.

MSC: 35B10; 35B32

Keywords: Vortex filaments; Periodic solutions; Bifurcation

0. Introduction

In [15] is derived a model for the movement of almost-parallel vortex filaments from the three-dimensional Euler equation. This model takes in consideration the interaction between different filaments and an approximation for the self-induction of each filament. The paper [15] presents a first analysis of the finite time collapse of two filaments with negative circulations; close to collapse, the model of vortex filaments as an approximation to the Euler equation loses validity. Later, [11] proves that two filaments with positive circulations, and also three filaments with positive circulations near an equilateral triangle, evolve without collapse for all time. On the

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other hand, the evidence shown in [2,3,15,16] suggests that two filaments with opposite circulations develop collapse for many initial configurations. Our aim is to investigate the existence of nontrivial periodic solutions of two vortex filaments with opposite circulations, which evolve without collapse and remains valid within the hypothesis of the model for all time.

The counter-rotating filament pair consists of two filaments with opposite circulations and same strength. In the model deduced in [15], the almost parallel filaments are parameterized by

$$(u_j(t,s), s) \in \mathbb{C} \times \mathbb{R}, \qquad j = 1, 2,$$

and the distance among the filaments $w_1 = u_1 - u_2$ satisfies the beam-type of equation

$$\partial_t^2 w_1 = -\partial_s^4 w_1 + \partial_s^2 \left(|w_1|^{-2} w_1 \right). \tag{1}$$

This equation has the explicit solution $w_1(t,s) = a$ that corresponds to the solution of two straight filaments traveling with speed a^{-1} at distance a. The aim is to construct $2\pi/\nu$-periodic families of standing wave bifurcating from this initial configuration, where the filaments have 2π-periodic boundary condition.

The present paper adopts the strategy followed in [12] for the wave equation, where bifurcation of periodic solutions is proven to exist using external parameters such as the amplitude, while the frequency is a fixed rational. In [13] and [18] this result was improved to obtain global bifurcation of periodic solutions in spherical domains. A main difference with our result is that the equation is semilinear and requires special estimates.

Theorem 1. *For each number q, there is an infinite number of non-resonant (Definition 7) amplitudes a_0's given by*

$$a_0^{-2} := (-1)^{l_0} \left(k_0^2 - (p/qk_0)^2 \right) \in (0, 1/q) \tag{2}$$

for some $p \in \mathbb{N}$, $k_0 \in \mathbb{N}$ and $l_0 \in \{0,1\} = \mathbb{Z}_2$. For each of these non-resonant a_0's, there is a local continuum of $2\pi q/p$-periodic solution bifurcating from the straight filaments with distance a_0. The local bifurcation consists of standing waves satisfying the symmetries

$$\begin{aligned} w_1(t,s) &= w_1(-t,s) = w_1(t,-s) = w_1(t, s + 2\pi/k_0) \\ &= \bar{w}_1\left(t + l_0(q\pi/p), s\right), \end{aligned} \tag{3}$$

and the estimate

$$w_1(t,s) = a_0 + i^{l_0} b \cos(pt/q) \cos k_0 s + \mathcal{O}_{C^4}(b^2), \tag{4}$$

where $b \in [0, b_0]$ gives a parameterization of the local bifurcation.

The symmetries imply that the standing waves are even in t and even and $2\pi/k_0$-periodic in s. Setting $w_1 = x + iy$, for $l_0 = 0$, the symmetry (3) implies that $y(t,s) = 0$, i.e. the orbits of the standing waves are orthogonal to the traveling direction of the filaments. While for $l_0 = 1$, this symmetry implies that