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Quenching rate for a nonlocal problem arising in the micro-electro mechanical system ☆

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Abstract

In this paper, we study the quenching rate of the solution for a nonlocal parabolic problem which arises in the study of the micro-electro mechanical system. This question is equivalent to the stabilization of the solution to the transformed problem in self-similar variables. First, some a priori estimates are provided. In order to construct a Lyapunov function, due to the lack of time monotonicity property, we then derive some very useful and challenging estimates by a delicate analysis. Finally, with this Lyapunov function, we prove that the quenching rate is self-similar which is the same as the problem without the nonlocal term, except the constant limit depends on the solution itself.

Keywords: Quenching; Micro-electro mechanical system (MEMS); Lyapunov function; Non-local; Self-similar; Asymptotic

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1. Introduction

In this paper, we consider the following initial boundary problem

$$u_t = u_{xx} - g(t; u, \lambda)u^{-2}, \quad -1 < x < 1,\ t > 0, \tag{1.1}$$

$$u(\pm 1, t) = 1, \quad t > 0, \tag{1.2}$$

$$u(x, 0) = u_0(x), \quad x \in [-1, 1], \tag{1.3}$$

where

$$g(t; u, \lambda) := \lambda\Big(1 + \int_{-1}^{1} u^{-1}(\xi, t)d\xi\Big)^{-2}. \tag{1.4}$$

Throughout this paper, we always assume that

$$\begin{aligned} &u_0 \text{ is smooth}, \quad u_0(\pm 1) = 1, \quad 0 < u_0(x) \leqslant 1, \quad u_0(x) = u_0(-x), \\ &u_0'(x) \geqslant 0, \quad u_0''(x) \geqslant 0 \quad \text{for } 0 \leqslant x \leqslant 1. \end{aligned} \tag{1.5}$$

Also, we shall simply denote $g(t; u, \lambda)$ by $g(t)$ when there is no confusion.

The problem (1.1)–(1.3) arises in the study of the micro-electro mechanical system. We refer to [27,28] for the physical background of this model. In fact, equation (1.1) is a special case of the following general model

$$\varepsilon u_{tt} + u_t = \Delta u - \frac{\lambda f(x)}{u^2\Big(1 + \alpha\int_{\Omega} u^{-1}(\xi, t)d\xi\Big)^2}, \quad x \in \Omega,\ t > 0, \tag{1.6}$$

where u represents the distance of the membrane and the ground electrode plate, ε is the ratio of the interaction due to the inertial and damping terms, λ is the applied voltage, $\alpha \geqslant 0$ is related to the capacitor and $f(x)$ is the varying dielectric properties of the membrane. The model (1.6) has been studied extensively, see, e.g., [20,7–9,17,21–23,25,19] for the case $\varepsilon = 0$ (without inertia) and [24,18] for the case $\varepsilon > 0$. We also refer the reader to a recent survey paper [16] for more details and some open problems.

It is known [17] that

Theorem 1. *Let (1.5) hold. Then*

(a) the system (1.1)–(1.3) admits a unique classical solution in the maximal existence interval $[0, T)$, i.e., for any small $\delta > 0$, the solution is in the class $u \in C^{2+\alpha,(2+\alpha)/2}([-1, 1] \times [0, T - \delta])$, $\min_{|x|\leqslant 1, 0\leqslant t\leqslant T-\delta} u(x, t) > 0$; furthermore, either $T = \infty$, or $0 < T < \infty$;
(b) for λ suitably large, the maximal existence interval $[0, T)$ is finite, i.e., solution $u(x, t)$ of (1.1)–(1.3) quenches in finite time $t = T$, and $u(0, t) = \min_{|x|\leqslant 1} u(x, t) \to 0$ as $t \to T^-$. Moreover, $x = 0$ is the only quenching point.

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