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Nonlinear fractional magnetic Schrödinger equation: Existence and multiplicity ☆

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Abstract

In this paper we focus our attention on the following nonlinear fractional Schrödinger equation with magnetic field

$$\varepsilon^{2s}(-\Delta)^s_{A/\varepsilon}u + V(x)u = f(|u|^2)u \quad \text{in } \mathbb{R}^N,$$

where $\varepsilon > 0$ is a parameter, $s \in (0, 1)$, $N \geq 3$, $(-\Delta)^s_A$ is the fractional magnetic Laplacian, $V : \mathbb{R}^N \to \mathbb{R}$ and $A : \mathbb{R}^N \to \mathbb{R}^N$ are continuous potentials and $f : \mathbb{R}^N \to \mathbb{R}$ is a subcritical nonlinearity. By applying variational methods and Ljusternick–Schnirelmann theory, we prove existence and multiplicity of solutions for ε small.
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1. introduction

In this paper we consider the following fractional nonlinear Schrödinger equation

$$\varepsilon^{2s}(-\Delta)^s_{A/\varepsilon}u + V(x)u = f(|u|^2)u \quad \text{in } \mathbb{R}^N \tag{1.1}$$

where $\varepsilon > 0$ is a parameter, $s \in (0,1)$, $N \geq 3$, $V \in C(\mathbb{R}^N, \mathbb{R})$ and $A \in C^{0,\alpha}(\mathbb{R}^N, \mathbb{R}^N)$, $\alpha \in (0,1]$, are the electric and magnetic potentials respectively, $u \in \mathbb{R}^N \to \mathbb{C}$, $f : \mathbb{R} \to \mathbb{R}$. The fractional magnetic Laplacian is defined by

$$(-\Delta)^s_A u(x) := c_{N,s} \lim_{r\to 0} \int\limits_{B^c_r(x)} \frac{u(x) - e^{\imath(x-y)\cdot A(\frac{x+y}{2})}u(y)}{|x-y|^{N+2s}} dy, \quad c_{N,s} := \frac{4^s\Gamma\left(\frac{N+2s}{2}\right)}{\pi^{N/2}|\Gamma(-s)|}. \tag{1.2}$$

This nonlocal operator has been defined in [16] as a fractional extension (for an arbitrary $s \in (0,1)$) of the magnetic pseudorelativistic operator, or *Weyl pseudodifferential operator defined with mid-point prescription*,

$$\begin{aligned}\mathscr{H}_A u(x) &= \frac{1}{(2\pi)^3} \int\limits_{\mathbb{R}^6} e^{\imath(x-y)\cdot\xi} \sqrt{\left|\xi - A\left(\frac{x+y}{2}\right)\right|^2} u(y)\,dy\,d\xi \\ &= \frac{1}{(2\pi)^3} \int\limits_{\mathbb{R}^6} e^{\imath(x-y)\cdot\left(\xi + A(\frac{x+y}{2})\right)} \sqrt{|\xi|^2} u(y)\,dy\,d\xi,\end{aligned}$$

introduced in [28] by Ichinose and Tamura, through oscillatory integrals, as a *fractional relativistic* generalization of the magnetic Laplacian (see also [27] and the references therein). Observe that for smooth functions u,

$$\begin{aligned}\mathscr{H}_A u(x) &= -\lim_{\varepsilon\searrow 0} \int\limits_{B^c_\varepsilon(0)} \left[e^{-\imath y\cdot A(x+\frac{y}{2})}u(x+y) - u(x) - 1_{\{|y|<1\}}(y)\,y\cdot(\nabla - \imath A(x))u(x)\right] d\mu \\ &= \lim_{\varepsilon\searrow 0} \int\limits_{B^c_\varepsilon(x)} \left[u(x) - e^{\imath(x-y)\cdot A\left(\frac{x+y}{2}\right)}u(y)\right] \mu(y-x)\,dy,\end{aligned}$$

where

$$d\mu = \mu(y)dy = \frac{\Gamma\left(\frac{N+1}{2}\right)}{\pi^{\frac{N+1}{2}}|y|^{N+1}} dy.$$

For details about the consistency of the definition in (1.2) we refer the reader to [32,34,35,39].

The study of nonlinear fractional Schrödinger equations attracted a great attention, specially in the case $A = 0$ (see [31] and references therein). For instance, Felmer et al. [21] dealt with existence, regularity and symmetry of positive solutions when V is constant, and f is a superlinear function with subcritical growth; see also [3,4,6,18] and [15] for the nonlocal Choquard equation. Secchi [37] obtained the existence of ground state solutions under the assumptions that

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