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Dynamics for a diffusive prey–predator model with different free boundaries ☆

Mingxin Wang [a,*], Yang Zhang [b]

[a] *Natural Science Research Center, Harbin Institute of Technology, Harbin 150080, PR China*
[b] *Department of Mathematics, Harbin Engineering University, Harbin 150001, PR China*

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Abstract

To understand the spreading and interaction of prey and predator, in this paper we study the dynamics of the diffusive Lotka–Volterra type prey–predator model with different free boundaries. These two free boundaries, which may intersect each other as time evolves, are used to describe the spreading of prey and predator. We investigate the existence and uniqueness, regularity and uniform estimates, and long time behaviors of global solution. Some sufficient conditions for spreading and vanishing are established. When spreading occurs, we provide the more accurate limits of (u, v) as $t \to \infty$, and give some estimates of asymptotic spreading speeds of u, v and asymptotic speeds of g, h. Some realistic and significant spreading phenomena are found.

MSC: 35K51; 35R35; 92B05; 35B40

Keywords: Diffusive prey–predator model; Different free boundaries; Spreading and vanishing; Long time behavior; Asymptotic propagation

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* Corresponding author.
E-mail addresses: mxwang@hit.edu.cn (M. Wang), zhangyanghit0217@163.com (Y. Zhang).

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1. Introduction

The spreading and vanishing of multiple species is an important content in understanding ecological complexity. In order to study the spreading and vanishing phenomenon, many mathematical models have been established. In this paper we consider the diffusive Lotka–Volterra type prey–predator model with different free boundaries. It is a meaningful subject, because the following phenomenon will happen constantly in the real world:

There is some kind of species (the indigenous species, prey u) in a bounded area (initial habitat, for example, Ω_0), and at some time (initial time, $t = 0$) another kind of species (the new or invasive species, predator v) enters a part Σ_0 of Ω_0.

In general, both species have tendencies to emigrate from boundaries to obtain their respective new habitats. That is, as time t increases, Ω_0 and Σ_0 will evolve into expanding regions $\Omega(t)$ and $\Sigma(t)$ with expanding fronts $\partial\Omega(t)$ and $\partial\Sigma(t)$, respectively. The initial functions $u_0(x)$ and $v_0(x)$ will evolve into positive functions $u(t, x)$ and $v(t, x)$ governed by a suitable diffusive prey–predator system, $u(t, x)$ and $v(t, x)$ vanish on the moving boundaries $\partial\Omega(t)$ and $\partial\Sigma(t)$, respectively. We want to understand the dynamics/variations of these two species and free boundaries. For simplicity, we assume that the interaction between these two species obeys the Lotka–Volterra law, and restrict our problem to the one dimensional case. Moreover, we think that the left boundaries of $\Omega(t)$ and $\Sigma(t)$ are fixed and coincident. So, we can take $\Omega_0 = (0, g_0)$, $\Sigma_0 = (0, h_0)$ with $0 < h_0 \le g_0$, and $\Omega(t) = (0, g(t))$, $\Sigma(t) = (0, h(t))$. Based on the *deduction of free boundary conditions* given in [3,41], we have the following free boundary conditions

$$g'(t) = -\beta u_x(t, g(t)), \quad h'(t) = -\mu v_x(t, h(t)),$$

where positive constants $\beta = d_1 k_1^{-1}$ and $\mu = d_2 k_2^{-1}$ can be considered as the *moving parameters*, d_1, d_2 and k_1, k_2 are, respectively, their diffusion coefficients and *preferred density levels* nearing free boundaries. Under the suitable rescaling, the model we are concerned here becomes the following free boundary problem

$$\begin{cases} u_t - du_{xx} = u(a - u - bv), & t > 0, \ \ 0 < x < g(t), \\ v_t - v_{xx} = v(1 - v + cu), & t > 0, \ \ 0 < x < h(t), \\ u_x(t, 0) = v_x(t, 0) = 0, & t \ge 0, \\ g'(t) = -\beta u_x(t, g(t)), \ \ h'(t) = -\mu v_x(t, h(t)), & t \ge 0, \\ u(t, x) = 0 \ \text{ for } x \ge g(t), \ \ v(t, x) = 0 \ \text{ for } x \ge h(t), & t \ge 0, \\ u(0, x) = u_0(x) \ \text{ in } [0, g_0], \ \ v(0, x) = v_0(x) \ \text{ in } [0, h_0], & \\ g(0) = g_0 \ge h_0 = h(0) > 0, & \end{cases} \tag{1.1}$$

where $a, b, c, d, g_0, h_0, \beta$ and μ are given positive constants. The initial functions $u_0(x)$, $v_0(x)$ satisfy

$$\begin{cases} u_0 \in C^2([0, g_0]), \ u_0'(0) = 0, \ u_0(x) > 0 \ \text{ in } [0, g_0), \ \ u_0(x) = 0 \ \text{ in } [g_0, \infty), \\ v_0 \in C^2([0, h_0]), \ v_0'(0) = 0, \ v_0(x) > 0 \ \text{ in } [0, h_0), \ \ v_0(x) = 0 \ \text{ in } [h_0, \infty). \end{cases} \tag{1.2}$$

Because the two free boundaries may intersect each other, it seems very difficult to understand the whole dynamics of this model. We shall see that the problem (1.1) possesses the multiplicity

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