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Existence and the dynamical behaviors of the positive solutions for a ratio-dependent predator–prey system with the crowing term and the weak growth ☆

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Abstract

This paper deals with a ratio-dependent predator–prey system with the crowing term and the weak growth in the prey equation. Under the condition that the coefficient λ is less than a critical value $\lambda_1^D(\Omega_0)$, we obtain existence of multiple positive steady state solutions of the predator–prey system and the dynamical behaviors of its positive solutions. Our results show that the predator and the prey possess not only the common coexistence, but also the very weak coexistence which both of the predator and the prey are very low. Meantime, the persistence of the positive solutions for the corresponding parabolic type system sometime depends strictly on the ratio of its initial data. Therefore, our results may be used to explain some special phenomena which under some bad environment, the predator and the prey may still coexist.

Keywords: Ratio-dependent predator–prey system; Crowding effect; Multiple positive solutions; Persistence; Stability

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1. Introduction

The dynamical relationships between the predators and their preys always are one of the dominant themes in the ecology and the mathematical ecology due to its universal existence and importance. Although these problems may appear to be some simple mathematical models, they often are very challenging and complicated. Furthermore, the mechanisms of some biological and ecological phenomena can depend strictly on some special mathematical models. To our knowledge, the classical prey-dependent predator–prey models exhibit the "paradox of enrichment" and the so-called "biological control paradox" [1,2,26]; but the ratio-dependent predator–prey model produces neither a paradox of enrichment nor the biological control paradox [30–32]. Therefore, the ratio-dependent predator–prey model should be a more reasonable model in the prey-dependent predator–prey models.

In this paper, we will investigate the existence and the dynamical behaviors of the positive solutions for the following predator–prey system

$$\begin{cases} u_t - d_1\Delta u = \lambda u - a(x)u^2 - \frac{buv}{u+mv}, & x \in \Omega, \quad t > 0, \\ v_t - d_2\Delta v = \mu v - v^2 + \frac{cuv}{u+mv}, & x \in \Omega, \quad t > 0, \\ \partial_\nu u = \partial_\nu v = 0, & x \in \partial\Omega, \quad t > 0, \\ u(x,0) = u_0(x), \quad v(x,0) = v_0(x), & x \in \overline{\Omega}, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbf{R}^N$ with smooth boundary $\partial\Omega$, $N \geq 1$, ν is the outward unit normal on $\partial\Omega$, and $\partial_\nu := \partial/\partial\nu$; $\lambda, \mu, b, c, m, d_1$ and d_2 are constants, and all of these constants are positive except μ which may take negative values; and $\lambda < b$ indicates that the prey is with the weak growth. $a(x)$ is a nonnegative continuous function in $\overline{\Omega}$. Moreover, there exists a subregion $\Omega_0 \subset \Omega$, such that $a(x) \equiv 0$ in $\overline{\Omega}_0$ and $a(x) > 0$ in $\overline{\Omega} \setminus \overline{\Omega}_0$, which implies that the prey species can be with the crowing term in Ω_0. We assume that Ω_0 is an open and connected subset of Ω, and $\partial\Omega_0 \in C^2$. The homogeneous Neumann boundary conditions indicate that System (1.1) is self-contained with zero population flux across the boundary. For the meanings of other terms and coefficients in (1.1), one can refer to the references [49,50,52,60,61].

System (1.1) arises in mathematical biology as a ratio-dependent predator–prey model of two species which are interacting each other and migrating in the same habitat Ω. In addition, the coefficient $a(x)$ in (1.1) is not constant, but a function of the space variable x. As far as we know, in the spatial population models, on account of the effect of the environment, some coefficients such as the growth rates, the crowding effects and the population interaction rates, are usually replaced by functions of the space variable x. The spatially heterogeneous models are very meaningful and valuable in the control to the alien species and the protection zone *etc.* [17–23,33–36,45,57,58,62,63]. Many pioneers such as Professors H. Brézis and L. Oswald [5], Cantrell and Cosner [6], Cirstea and Radulescu [7–9], Du and his coauthors [17–25], J.M. Fraile and his coauthors [27], J. García-Melián and his coauthors [28,29], López-Gómez and his coauthors [3,28,33–41] and Ouyang [48] *etc.*, have many outstanding fundamental works in this field. For the class of population models in a spatially heterogeneous environment, it has been observed that in general, the behaviors of the solutions of the class of population models are very sensitive to the change of certain coefficient functions in part of the underlying spatial region, and the coexistence of two species depends on the relationship of the growth rates λ with the principal eigenvalue $\lambda_1^D(\Omega_0)$. We find that these results of [3,17–23,25,33–36,45,57,58,62,63]