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Journal of Differential Equations

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# Local controllability to stationary trajectories of a Burgers equation with nonlocal viscosity

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Received 5 July 2017; revised 22 November 2017

**Abstract**

This article studies the local controllability to trajectories of a Burgers equation with nonlocal viscosity. By linearization we are led to an equation with a non local term whose controllability properties are analyzed by using Fourier decomposition and biorthogonal techniques. Once the existence of controls is proved and the dependence of their norms with respect to the time is established for the linearized model, a fixed point method allows us to deduce the result for the nonlinear initial problem.


*MSC:* 93C20; 93B05; 93B07; 35Q53; 35P20

*Keywords:* Controllability to trajectories; Viscous Burgers equation; Series of exponentials; Observability; Biorthogonal

**Contents**



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https://doi.org/10.1016/j.jde.2017.11.029

## 1. Introduction

In [10], the authors tackle the local null controllability of a fluid model introduced by Ladyzhenskaya (see [16], [17, p. 193], [18, p. 208]). This system is similar to the classical Navier–Stokes system but with a viscosity depending on the total dissipation energy of the fluid. The study in [10] deals with the following control problem

$$\begin{cases} \partial_t w + (w\cdot\nabla)w - \nu(\|\nabla w\|^2_{L^2(\Omega)})\Delta w + \nabla q = u\chi_\omega & \text{in } (0,T)\times\Omega, \\ \operatorname{div} w = 0 & \text{in } (0,T)\times\Omega, \\ w = 0 & \text{on } (0,T)\times\partial\Omega \\ w(0,\cdot) = w_0 & \text{in } \Omega, \end{cases}$$

where $\nu : \mathbb{R}_+ \to [\nu_0, \infty)$ is a $C^1$ function, with $\nu_0 > 0$ and with bounded derivatives. In the above system, $\Omega$ is a bounded domain of $\mathbb{R}^d$, $d = 2, 3$, and $w$ and $q$ are respectively the velocity and the pressure of the fluid. The control $u = u(t, x)$ acts on a part $\omega$ of the domain. Following the method used for the controllability of the Navier–Stokes system (see [9]), they show a Carleman estimate for the linearized system and, with a fixed point argument, they obtain the local controllability to the null state ($w(T,\cdot) \equiv 0$). In Section 6 of [10], it is mentioned that at the contrary to the Navier–Stokes system, it is not clear how to obtain the local controllability to the trajectories. Indeed, in the linearized system, one has to deal with non-local terms.

In [11], a partial answer to the above question is given: the authors consider the linear heat and the linear wave system and show that one can recover the controllability properties of these systems if some non local terms are added. More precisely, their results are obtained for any dimension in space, but they need that the nonlocal integral terms are analytic in space.

Unhappily, in the above paper, the proof is given through a contradiction argument and in particular, it is not clear how to keep the cost of the linear heat equation in order to tackle nonlinear problems. Our aim is here to study the controllability of a Burgers system with a nonlocal viscosity. This can be seen as a simplified model of the model considered in [10]. More precisely, our system writes as

$$\begin{cases} \partial_t w - \nu(\|\partial_x w\|^2_{L^2(0,\pi)})\partial_{xx} w + w\partial_x w = f^S + u\chi_\omega & \text{in } (0,T)\times(0,\pi), \\ w(t,0) = w(t,\pi) = 0 & t\in(0,T), \\ w(0,\cdot) = w_0 & \text{in } (0,\pi), \end{cases} \tag{1.1}$$

Please cite this article in press as: S. Micu, T. Takahashi, Local controllability to stationary trajectories of a Burgers equation with nonlocal viscosity, J. Differential Equations (2017), https://doi.org/10.1016/j.jde.2017.11.029