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ScienceDirect
J. Differential Equations 264 (2018) 3704–3774
Journal of Differential Equations
www.elsevier.com/locate/jde

Quadratic obstructions to small-time local controllability for scalar-input systems

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Received 22 May 2017; revised 9 October 2017
Available online 6 December 2017

Abstract

We consider nonlinear finite-dimensional scalar-input control systems in the vicinity of an equilibrium.

When the linearized system is controllable, the nonlinear system is smoothly small-time locally controllable: whatever $m > 0$ and $T > 0$, the state can reach a whole neighborhood of the equilibrium at time T with controls arbitrary small in C^m-norm.

When the linearized system is not controllable, we prove that: either the state is constrained to live within a smooth strict manifold, up to a cubic residual, or the quadratic order adds a signed drift with respect to it. This drift holds along a Lie bracket of length $(2k + 1)$, is quantified in terms of an H^{-k}-norm of the control, holds for controls small in $W^{2k,\infty}$-norm and these spaces are optimal. Our proof requires only C^3 regularity of the vector field.

This work underlines the importance of the norm used in the smallness assumption on the control, even in finite dimension.

MSC: 93B05; 93C15; 93B10

Keywords: Controllability; Quadratic expansion; Scalar-input; Obstruction; Small-time

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https://doi.org/10.1016/j.jde.2017.11.028
0022-0396/

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