



# Quadratic obstructions to small-time local controllability for scalar-input systems

Karine Beauchard <sup>a,\*</sup>, Frédéric Marbach <sup>b,\*</sup>

<sup>a</sup> IRMAR, Ecole Normale Supérieure de Rennes, UBL, CNRS, Campus de Ker Lann, 35170 Bruz, France

<sup>b</sup> Sorbonne Universités, UPMC Université Paris 06, CNRS UMR 7598, Laboratoire Jacques-Louis Lions, Place Jussieu, 75005 Paris, France

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## Abstract

We consider nonlinear finite-dimensional scalar-input control systems in the vicinity of an equilibrium.

When the linearized system is controllable, the nonlinear system is smoothly small-time locally controllable: whatever  $m > 0$  and  $T > 0$ , the state can reach a whole neighborhood of the equilibrium at time  $T$  with controls arbitrary small in  $C^m$ -norm.

When the linearized system is not controllable, we prove that: either the state is constrained to live within a smooth strict manifold, up to a cubic residual, or the quadratic order adds a signed drift with respect to it. This drift holds along a Lie bracket of length  $(2k + 1)$ , is quantified in terms of an  $H^{-k}$ -norm of the control, holds for controls small in  $W^{2k, \infty}$ -norm and these spaces are optimal. Our proof requires only  $C^3$  regularity of the vector field.

This work underlines the importance of the norm used in the smallness assumption on the control, even in finite dimension.

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\* Corresponding authors.

E-mail addresses: [karine.beauchard@ens-rennes.fr](mailto:karine.beauchard@ens-rennes.fr) (K. Beauchard), [frederic.marbach@upmc.fr](mailto:frederic.marbach@upmc.fr) (F. Marbach).

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