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Controllability of a multichannel system

Sergei A. Ivanov [a,*], Jun Min Wang [b]

[a] *Marine Geomagnetic Investigation Laboratory, SPbF IZMIRAN, Mendeleev line 1, 199034, St. Petersburg, Russia*
[b] *School of Mathematics and Statistics, Beijing Institute of Technology, Beijing 100081, P. R. China*

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Abstract

We consider the system consisting of K coupled acoustic channels with the different sound velocities c_j. Channels are interacting at any point via the pressure and its time derivatives. Using the moment approach and the theory of exponential families with vector coefficients we establish two controllability results: the system is exactly controllable if

(i) the control u_j in the jth channel acts longer than the double travel time of a wave from the start to the end of the j-th channel;

(ii) all controls u_j act more than or equal to the maximal double travel time.

MSC: 45K05; 35P20

Keywords: Controllability; Nonharmonic Fourier series Riesz basis

1. Introduction and main results

We study the control system governed by the differential equation

$$y_{tt}(x,t) = Dy_{xx}(x,t) + Q(x)y(x,t) + R(x)y_t(x,t), \ x \in (0,1), \tag{1}$$

* Corresponding author.
E-mail addresses: sergei.a.ivanov@mail.ru (S.A. Ivanov), jmwang@bit.edu.cn (J.M. Wang).

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where D is a diagonal $K \times K$ matrix, $D = \mathrm{diag}[c_j^2]_1^K$, $c_j > 0$, with different entries c_j, $c_i \neq c_j$, for $i \neq j$, Q and R are C^1-smooth matrix valued functions. The controls act in the channels at the boundary entering the Dirichlet boundary condition (BC) at one end. At the other end we have a homogeneous BC

$$y(0,t) = u(t), \; y(1,t) = 0. \tag{2}$$

All channel controls u_j are quadratically integrable at any interval.

The unperturbed system with $R = Q = 0$ represents K uncoupled channels governed by the string equation with constant coefficients. Such systems are controllable because each channel is exact controllable in time $T_i = 2/c_i$, this is the double travel time of a wave from the start to the end of the j-th channel.

We show that the system is exactly controllable in the state space $L^2(0,1;\mathbb{C}^K) \oplus H^{-1}(0,1;\mathbb{C}^K)$ for the following types of controls:

Type (i): for all j the control in the jth channel acts longer than T_j,

or

Type (ii): the whole control u acts longer than or equal to $T_{max} = \max T_j$ what is the double maximal travel time.

The first paper on controllability of a coupled system, namely, of a connected network of homogeneous strings with controls at the nodes, was written by Rolewicz [16]. In this statement the string network controllability problem was completely solved in [2]. Now many strong results in theory of control and stabilization of networks are obtained, see, for example, [20,21,1].

The main feature of the system (1), (2) is the presence of different wave modes propagate with different velocities and interact with each other. For the case of one unit velocity the controllability results are similar to an 1dim system. In [3] the control system governed by the equation with potential was studied

$$\begin{aligned} y_{tt}(x,t) = &\quad y_{xx}(x,t) + V(x)y(x,t), \; x,t > 0 \; y(x,t) \in \mathbb{C}^K, \\ y|_{t<0} = 0, &\quad y(0,t) = u(t). \end{aligned}$$

The *shape–controllability in the filled domain* was proved: for u runs $L^2(0,T;\mathbb{C}^K)$ the first component $y(\cdot,T)$ of the state runs exactly $L^2(0,T;\mathbb{C}^K)$. If the system is considered on an spatial interval $(0,L)$ with the Dirichlet BC (2), we have controllability of Type (ii) in time $T = L$ as in the case of uncoupled channels.

In [19] an interesting case of the system of parallel coupled identical strings was considered. The authors prove the controllability and show that despite the damping, there exist sustained oscillations.

More interesting dynamic and more difficult control problems arise in the system possessing two-mode oscillations. In a series of papers, see, e.g., [4,5] the dynamic and the shape–controllability in the filled domain are studied. A classical example of a two-channel system with interacting modes is a Timoshenko beam in elasticity theory. In [9,17] the system with a distributed (in the right hand side of the equation) '1dim' control of the form $u(t)\binom{r_1(x)}{r_2(x)}$ is studied. Here r_1 and r_2 are fixed functions. To obtain the controllability the authors use a non-physical condition that the velocities c_1 and c_2 are commensurable: c_1/c_2 is a rational number. The reason is that the authors consider the conservative BC and apply the moment approach what leads to a moment problem with respect to exponential family $exp(ik_nt)$ where the real set of