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Global BV solution for a non-local coupled system modeling the dynamics of dislocation densities

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Abstract

In this paper, we study a non-local coupled system arising in the modeling of the dynamics of dislocation densities in crystals. For this system, the global existence and uniqueness are available only for continuous viscosity solutions. In the present paper, we investigate the global time existence of this system by considering BV initial data. Based on a fundamental uniform BV estimate and the finite speed of propagation property of this system, we show, in a particular setting, the global existence of discontinuous viscosity solutions of this problem.

MSC: 35Q53; 35Q72; 49L25; 35F25; 35L40; 35L45; 65M06; 65M12; 65M15; 74H20; 74H25

Keywords: Hamilton–Jacobi system; Non-local eikonal system; Non-local transport system; BV estimate; Discontinuous viscosity solution; Dislocation dynamics

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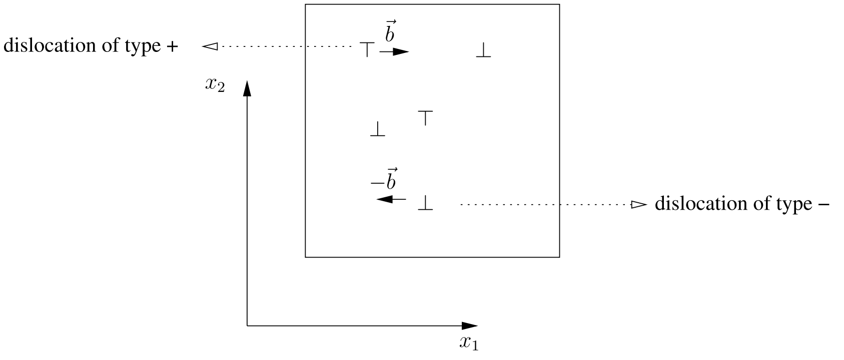

Fig. 1. The cross-section of the dislocations lines.

1. Introduction and main results

1.1. Presentation and physical motivations

A dislocation is a crystal defect which corresponds to a discontinuity in the crystalline structure organization. This concept has been introduced by Polanyi, Taylor and Orowan in 1934 as the main explanation, at the microscopic scale, of plastic deformation. A dislocation creates around it a perturbation that can be seen as an elastic field. Under an exterior strain, a dislocation moves according to its Burgers vector which characterizes the intensity and the direction of the defect displacement (see Hirth and Lothe [1] for an introduction to dislocations).

Here, we are interested in the dynamics of dislocation densities. More precisely, we consider a particular type called the edge dislocations where the line holding the dislocations is perpendicular to the Burgers vector. In a simple geometric setting where dislocation lines are assumed to be parallel, it can be seen that the dislocation distribution in any 2-d plane, perpendicular to these lines, are as shown in Fig. 1. In this case, the dislocation points are moving along with the Burgers vectors $\pm\vec{b}$, hence it is convenient to distinguish two types of dislocations, the positive (following $+\vec{b}$) and the negative (following $-\vec{b}$) dislocations.

In this model (see Fig. 1), the dynamics of the dislocation densities has been introduced by Groma and Balogh [2] as a coupled system, namely a transport problem where the velocity is given by the elasticity equations.

We assume $\vec{b} = (1, 0)$. If the 2-d domain is 1-periodic in x_1 and x_2, and if the dislocation densities depend only on the variable $x = x_1 + x_2$ (where (x_1, x_2) is the coordinate of a generic point in $\mathbb{R}^2$), then the 2-d model of [2] reduces to the following system of 1-d coupled non-local Hamilton–Jacobi (HJ for short) equations:

$$\begin{cases} \dfrac{\partial\rho^+}{\partial t}(x,t) = -\left((\rho^+ - \rho^-)(x,t) + \alpha\displaystyle\int_0^1 (\rho^+ - \rho^-)(y,t)dy + a(t)\right)\left|\dfrac{\partial\rho^+}{\partial x}(x,t)\right| & \text{in } \mathbb{R}\times(0,T), \\ \dfrac{\partial\rho^-}{\partial t}(x,t) = \left((\rho^+ - \rho^-)(x,t) + \alpha\displaystyle\int_0^1 (\rho^+ - \rho^-)(y,t)dy + a(t)\right)\left|\dfrac{\partial\rho^-}{\partial x}(x,t)\right| & \text{in } \mathbb{R}\times(0,T), \end{cases} \tag{1.1}$$

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