ARTICLE IN PRESS YJDEQ:9056

Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Positive effects of repulsion on boundedness in a fully parabolic attraction–repulsion chemotaxis system with logistic source ☆

Wei Wang *, Mengdi Zhuang, Sining Zheng

School of Mathematical Sciences, Dalian University of Technology, Dalian 116024, PR China

Received 24 July 2017; revised 4 October 2017

Abstract

In this paper we study the global boundedness of solutions to the fully parabolic attraction–repulsion chemotaxis system with logistic source: $u_t = \Delta u - \chi\nabla\cdot(u\nabla v) + \xi\nabla\cdot(u\nabla w) + f(u)$, $v_t = \Delta v - \beta v + \alpha u$, $w_t = \Delta w - \delta w + \gamma u$, subject to homogeneous Neumann boundary conditions in a bounded and smooth domain $\Omega \subset \mathbb{R}^n$ ($n \geq 1$), where χ, α, ξ, γ, β and δ are positive constants, and $f : \mathbb{R} \to \mathbb{R}$ is a smooth function generalizing the logistic source $f(s) = a - bs^\theta$ for all $s \geq 0$ with $a \geq 0$, $b > 0$ and $\theta \geq 1$. It is shown that when the repulsion cancels the attraction (i.e. $\chi\alpha = \xi\gamma$), the solution is globally bounded if $n \leq 3$, or $\theta > \theta_n := \min\left\{\frac{n+2}{4}, \frac{n\sqrt{n^2+6n+17}-n^2-3n+4}{4}\right\}$ with $n \geq 2$. Therefore, due to the inhibition of repulsion to the attraction, in any spatial dimension, the exponent θ is allowed to take values less than 2 such that the solution is uniformly bounded in time.

© 2017 Elsevier Inc. All rights reserved.

MSC: 35B35; 35B40; 35K55; 92C17

Keywords: Attraction–repulsion; Fully parabolic; Chemotaxis; Boundedness; Logistic source

☆ Supported by the National Natural Science Foundation of China (11671066, 11171048) and the Fundamental Research Funds for the Central Universities (DUT16LK24).

* Corresponding author.
E-mail address: weiwang@dlut.edu.cn (W. Wang).

https://doi.org/10.1016/j.jde.2017.10.011
0022-0396/© 2017 Elsevier Inc. All rights reserved.

Please cite this article in press as: W. Wang et al., Positive effects of repulsion on boundedness in a fully parabolic attraction–repulsion chemotaxis system with logistic source, J. Differential Equations (2017), https://doi.org/10.1016/j.jde.2017.10.011

1. Introduction

This paper is concerned with the global boundedness of solutions to the fully parabolic attraction–repulsion chemotaxis system with logistic source

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (u \nabla v) + \xi \nabla \cdot (u \nabla w) + f(u), & (x,t) \in \Omega \times (0,T), \\ v_t = \Delta v - \beta v + \alpha u, & (x,t) \in \Omega \times (0,T), \\ w_t = \Delta w - \delta w + \gamma u, & (x,t) \in \Omega \times (0,T), \\ \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = \frac{\partial w}{\partial \nu} = 0, & (x,t) \in \partial\Omega \times (0,T), \\ (u(x,0), v(x,0), w(x,0)) = (u_0(x), v_0(x), w_0(x)), & x \in \Omega \end{cases} \tag{1.1}$$

in a bounded and smooth domain $\Omega \subset \mathbb{R}^n$ $(n \geq 1)$, where χ, α, ξ, γ, β and δ are positive constants, and $f : \mathbb{R} \to \mathbb{R}$ is smooth and satisfies $f(0) \geq 0$ as well as

$$f(s) \leq a - bs^\theta \quad \text{for all } s \geq 0 \text{ with some } a \geq 0,\ b > 0 \text{ and } \theta \geq 1. \tag{1.2}$$

In addition, $\partial/\partial\nu$ represents the outer normal derivative on $\partial\Omega$, and the initial data $u_0 \in C^0(\bar{\Omega})$ and $v_0, w_0 \in W^{1,\infty}(\Omega)$ are nonnegative with $u_0 \not\equiv 0$.

Model (1.1) describes a biological process in which cells (with density u) exhibit two kinds of partially oriented movement in response to the chemical signals produced by themselves, namely, migrating towards higher concentrations of an attractive signal v (chemoattractant) and staying away from a repulsive signal w (chemorepellent) [11,20]. Also, the inhomogeneity $f(u)$, comprising a possible proliferation of cells and a growth restriction of logistic type due to (1.2), represents the cell kinetic mechanism.

The system (1.1) is a generalized version of the following Keller–Segel model without the repulsion

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (u \nabla v) + f(u), & (x,t) \in \Omega \times (0,T), \\ v_t = \Delta v - \beta v + \alpha u, & (x,t) \in \Omega \times (0,T), \\ \frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x),\ v(x,0) = v_0(x), & x \in \Omega. \end{cases} \tag{1.3}$$

Intuitively, the repulsion in (1.1) benefits the global boundedness of solutions. However, it seems that some challenges in the qualitative descriptions of (1.1) have to be confronted due to the lack of necessary Lyapunov functionals, and sometimes the blow-up prevention by the repulsion is reflected only in some special cases. As comparison, let us briefly recall related literature on (1.3), (1.1) and so forth.

(I) The case without growth source, viz. $f \equiv 0$

The solutions of (1.3) remain globally bounded when either $n = 1$, or $n = 2$ and $\int_\Omega u_0 < 4\pi/(\alpha\chi)$, or $n \geq 3$ and $\|u_0\|_{L^{n/2}(\Omega)} + \|\nabla v_0\|_{L^n(\Omega)}$ is sufficiently small [2,7,17,19,27]; whereas

Please cite this article in press as: W. Wang et al., Positive effects of repulsion on boundedness in a fully parabolic attraction–repulsion chemotaxis system with logistic source, J. Differential Equations (2017), https://doi.org/10.1016/j.jde.2017.10.011

Download English Version:

https://daneshyari.com/en/article/8899032

Download Persian Version:

https://daneshyari.com/article/8899032

Daneshyari.com