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Optimal decay estimates of a regularity-loss type system with constraint condition

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Abstract

In the paper [14], the authors formulated a new structural condition which includes the Kawashima–Shizuta condition, and analyzed the weak dissipative structure called the regularity-loss type for general systems which contain the Timoshenko system and the Euler–Maxwell system. However, this new structural condition can not cover all of dissipative systems. Indeed we introduce a dissipative system which does not satisfy the new condition and analyze the weaker dissipative structure in this paper. Precisely we first derive the L^2 decay estimate of solutions and discuss the type of the corresponding regularity-loss structure. Moreover, in order to show the optimality of the decay estimate, we analyze the expansion for the corresponding eigenvalue of our problem and derive that the solution approaches the diffusion wave as time tends to infinity.

MSC: 35B35; 35B40; 35C20

Keywords: Optimal decay estimate; Regularity-loss structure; Energy method in Fourier space; Eigenvalue expansion

1. Introduction

Our purpose of this paper is to analyze the dissipative structure for problems of the regularity-loss type. Recently we found some complicated physical models which possess the weak dissipative structure, which is called the regularity-loss structure. For example, the dissipative Timoshenko system was discussed in [3,4,12], the Euler–Maxwell system was studied in [1,17,18],

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and the hybrid problem of plate equations is in [7,8,10,11]. Moreover, Ueda–Duan–Kawashima in [14] tried to construct the new stability condition in order to analyze the regularity-loss structure for the general symmetric hyperbolic systems. However, the stability condition constructed in [14] is not completely enough to understand the regularity-loss structure. In fact, some physical models which possess the regularity-loss structure do not satisfy the stability condition in [14] (e.g. [11]). Furthermore, we can construct artificial models which have the several kinds of the regularity-loss structure (in detail, see [16]).

Under this situation, in order to construct the essential stability condition, we need more concrete examples which possess the regularity-loss structure and have to study more detailed properties. To this end, in this paper, we consider the Cauchy problem for a couple of wave and heat equations as one concrete example. Precisely, we treat the following Cauchy problem:

$$\begin{aligned} &u_{tt} - \Delta u + \gamma\theta = 0, \\ &\theta_t - \gamma u_t - \nu\Delta\theta = 0 \end{aligned} \tag{1.1}$$

with initial data

$$u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), \quad \theta(0,x) = \theta_0(x).$$

Here $u = u(t,x)$ and $\theta = \theta(t,x)$ over $t > 0$, $x \in \mathbb{R}^n$ are unknown scalar functions, $u_0 = u_0(x)$, $u_1 = u_1(x)$ and $\theta_0 = \theta_0(x)$ over $x \in \mathbb{R}^n$ are given scalar functions, and γ and ν denote constants which satisfy $\gamma \in \mathbb{R}\backslash\{0\}$ and $\nu > 0$. The system (1.1) is one of the typical examples of the regularity-loss type equations. Indeed this system was concerned in [6] and obtained the weak dissipative structure in a bounded domain. Moreover, Liu–Rao in [9] analyzed this equation to derive the stability criterion for the regularity-loss type problems in a bounded domain.

We rewrite (1.1) to get more formal representation. Introduce the new functions v and w as $v = \nabla u$ and $w = u_t$. Then (1.1) can be rewritten as

$$\begin{aligned} &v_t - \nabla w = 0, \\ &w_t - \operatorname{div} v + \gamma\theta = 0, \\ &\theta_t - \gamma w - \nu\Delta\theta = 0. \end{aligned} \tag{1.2}$$

We also define initial data for (1.2)

$$v(0,x) = v_0(x), \quad w(0,x) = w_0(x), \quad \theta(0,x) = \theta_0(x), \tag{1.3}$$

where $v_0(x) := \nabla u_0(x)$ and $w_0(x) := u_1(x)$. Here we remark that the solution v should satisfy $\partial_{x_j} v^k - \partial_{x_k} v^j = 0$ for an arbitrary j and k with $1 \le j, k \le n$, where v^j denotes the j-th component of the vector v. Thus we introduce the following constraint condition in this problem.

Constraint condition: Suppose the constraint condition for $\phi(x) = (\phi^1, \cdots, \phi^n)(x) \in \mathbb{R}^n$:

$$\partial_{x_j}\phi^k - \partial_{x_k}\phi^j = 0, \qquad 1 \le j, k \le n. \tag{1.4}$$

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