ARTICLE IN PRESS

YJDEQ:9022

Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Generalized Newtonian fluids in moving domains

Philipp Nägele, Michael Růžička *

Faculty of Mathematics and Physics, Department of Mathematics, University of Freiburg, Eckerstrasse 1, 79104 Freiburg i. Br., Germany

Received 21 July 2016; revised 4 September 2017

Abstract

In this paper we prove the existence of weak solutions for the equations describing the unsteady motion of an incompressible, viscous and homogeneous generalized Newtonian fluid in a non-cylindrical domain $\bigcup_{t\in I}\{t\}\times\Omega(t)$.

MSC: 76D03; 76A05; 74F10; 35R37; 46B50; 35K55

Keywords: Non-cylindrical domain; Existence of weak solutions; Nonlinear parabolic problem; Compactness

1. Introduction

In this paper we study the existence of weak solutions of the flow of an incompressible, viscous and homogeneous generalized Newtonian fluid which is contained in an impermeable moving space-time cylinder

$$Q=\bigcup_{t\in I}\{t\}\times\Omega(t)\subset\mathbb{R}^4$$

with moving boundary

* Corresponding author.
E-mail address: rose@mathematik.uni-freiburg.de (M. Růžička).

https://doi.org/10.1016/j.jde.2017.09.022

$$\Gamma = \bigcup_{t \in I} \{t\} \times \partial\Omega(t).$$

Given a force density $\mathbf{f}$, an initial velocity $\mathbf{u}_0$ and the boundary velocity $\mathbf{v}$, the motion of the fluid is modelled by means of the following generalized Navier–Stokes system for the velocity field $\mathbf{u}$ and the scalar pressure π:

$$\begin{aligned} \partial_t \mathbf{u} + \big(\mathbf{u} \cdot \nabla\big)\mathbf{u} - \operatorname{div} \mathbf{S}(\mathbf{D}\mathbf{u}) + \nabla \pi = \mathbf{f} \quad & \text{in } Q, \\ \operatorname{div} \mathbf{u} = 0 \quad & \text{in } Q, \\ \mathbf{u} = \mathbf{v} \quad & \text{on } \Gamma, \\ \mathbf{u}(0) = \mathbf{u}_0 \quad & \text{in } \Omega_0. \end{aligned} \tag{1}$$

There is a vast diversity of constitutive models characterizing generalized Newtonian fluids. Two commonly used constitutive relations for the nonlinear stress tensor $\mathbf{S}$, also known as power-law ansatz models, are given by

$$\begin{aligned} \mathbf{S}(\mathbf{D}\mathbf{u}) &:= \mu_0\big(\delta + |\mathbf{D}\mathbf{u}|\big)^{p-2}\mathbf{D}\mathbf{u}, \\ \mathbf{S}(\mathbf{D}\mathbf{u}) &:= \mu_0\big(\delta + |\mathbf{D}\mathbf{u}|^2\big)^{\frac{p-2}{2}}\mathbf{D}\mathbf{u}. \end{aligned} \tag{2}$$

Here, $\mu_0 > 0$, $\delta \geq 0$ and $1 < p < \infty$ are constants and $\mathbf{D}\mathbf{u}$ denotes the symmetric part of the velocity gradient $\nabla\mathbf{u}$. Note that the Dirichlet condition $(1)_3$ means that the fluid particles on the boundary move according to the velocity of the boundary. Note also that for $p = 2$, both relations in (2) reduce to the well-known Navier–Stokes model. Therefore, our existence result about weak solutions of (1) is a generalization of results in [1], [2] and [3], where the Newtonian case of the above system is treated. In what follows we always assume that the boundary velocity $\mathbf{v}$ is smooth with respect to space and time although it would be enough to require $\mathbf{v} \in C^2(\overline{Q})^3$. It is not realistic to assume this degree of smoothness if, as for fluid-structure-interaction (FSI) problems, the boundary velocity is determined by the solutions of the structure equations which are merely continuous in general. However, by regularizing the structure equations through adding appropriate damping terms one can gain a certain amount of smoothness, see for instance [4]. It is also possible to smooth the solution of the structure equation directly. For this approach we refer to [5]. Apart from being interesting in itself, the above system may therefore also serve as a first approximation for a subproblem of a fully coupled FSI problem for a generalized Newtonian fluid.

The aim of this paper is to prove existence of weak solutions of (1) up to the critical lower bound $p > 6/5$ in three space dimensions without transforming the system to an auxiliary system on a fixed cylindrical domain. Nevertheless we use the Piola transform to prove certain properties of the relevant function spaces and to construct appropriate ansatz function. Note that the Piola transform was already used in the context of FSI problems among others in [6], [7] and [5]. There are two crucial steps in the existence proof. On the one hand one has to identify the nonlinear elliptic term induced by the stress tensor $\mathbf{S}$. On the other hand, the passage to the limit in the approximate system and especially in the nonlinear convective term demands for global (non-cylindrical) compactness methods.

In the case $11/5 \leq p < \infty$, the identification of the elliptic term rests upon an integration-by-parts formula for functions with generalized time-derivative in the dual of the natural energy

Please cite this article in press as: P. Nägele, M. Růžička, Generalized Newtonian fluids in moving domains, J. Differential Equations (2017), https://doi.org/10.1016/j.jde.2017.09.022