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Journal of Differential Equations

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Quasi-neutral and zero-viscosity limits of Navier–Stokes–Poisson equations in the half-space

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Received 29 June 2017

Abstract

The present paper is concerned with the quasi-neutral and zero-viscosity limits of Navier–Stokes–Poisson equations in the half-space. We consider the Navier-slip boundary condition for velocity and Dirichlet boundary condition for electric potential. By means of asymptotic analysis with multiple scales, we construct an approximate solution of the Navier–Stokes–Poisson equations involving two different kinds of boundary layer, and establish the linear stability of the boundary layer approximations by conormal energy estimate.

MSC: 35Q35; 76N20; 76X05

Keywords: Navier–Stokes–Poisson equation; Quasi-neutral limit; Zero-viscosity limit; Boundary layer

1. Introduction

In this paper we consider the quasi-neutral and zero-viscosity limits for the Navier–Stokes–Poisson (NSP) equations in three dimensional half-space. For space–time variable $(t, x) = (t, x_1, x_2, x_3) = (t, y, x_3) \in \mathbb{R}_+ \times \mathbb{R}^2 \times \mathbb{R}_+$, the isothermal NSP equations take the following form

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https://doi.org/10.1016/j.jde.2017.09.021

Please cite this article in press as: Q. Ju, X. Xu, Quasi-neutral and zero-viscosity limits of Navier–Stokes–Poisson equations in the half-space, J. Differential Equations (2017), https://doi.org/10.1016/j.jde.2017.09.021

$$\begin{cases} \rho_t + \nabla \cdot (\rho u) = 0, \\ (\rho u)_t + \nabla \cdot (\rho u \otimes u + T^i \rho \mathbb{I}) = \rho \nabla \phi + \mu' \triangle u + (\mu' + \nu') \nabla \nabla \cdot u, \\ \lambda \triangle \phi + e^{-\phi} = \rho, \end{cases} \tag{1.1}$$

where the unknown functions ρ, u, ϕ are the density of the fluid, the velocity, electric potential, respectively. T^i is the average temperature of the ions. λ is a small parameter which represents the squared scaled Debye length. μ' and ν' are constant viscosity coefficients with $\mu' > 0$ and $\mu' + \nu' > 0$. This system is used to simulate the behavior of ions in a background of massless electrons.

We complete the system (1.1) with the following boundary conditions

$$u_3 = 0, \qquad u_i - \alpha \frac{\partial u_i}{\partial x_3} = 0, \qquad i = 1, 2, \qquad \phi = \phi_b, \tag{1.2}$$

on $x_3 = 0$, where $\phi_b = \phi_{\text{ref}} + \phi(y)$ with ϕ_{ref} being a constant and $\phi(y)$ is a smooth function. The boundary condition on u is the Navier-slip type boundary condition with $\alpha > 0$ being the slip length. This type boundary condition was introduced by Navier in [20] and expresses the condition that the velocity on the boundary is proportional to the tangential component of the stress. See [10] for an elementary derivation of the Navier boundary condition.

In the present paper, we are interested in the behavior of the system (1.1) with boundary condition (1.2) in the regime of small Debye length and zero-viscosity. Small Debye length is characteristic for the quasi-neutral limit. Recently, Donatelli, Feireisl and Novotný [4] studied such kind of limits for the weak solutions to the problem (1.1)–(1.2) of cold plasma. They analyzed the associated singular limits and identify the limit problem-incompressible Euler system. Moreover, acoustic oscillatory wave was involved in the solutions. However, quite different from their work, we are intended to investigate the singular limits (quasi-neutral and zero-viscosity) for the smooth solution to the system (1.1)–(1.2). The boundary layers develop due to the interaction between plasma and the boundary. Moreover two kinds of boundary layers will develop in the process of limits. One is Debye layer because the quasineutrality breaks down near the boundary. The other one is because of the zero viscosity limit around the boundary. We will see that the former is the strong boundary layer and the latter is the weak one.

For our purpose, we assume that

$$\mu' = \mu \epsilon^2, \qquad \nu' = \nu \epsilon^2, \qquad \lambda = \epsilon^2. \tag{1.3}$$

We consider the behavior of the solution when $\epsilon \to 0$.

Formally, letting $\epsilon = 0$ in (1.1), we immediately have

$$\begin{cases} \rho_t + \nabla \cdot (\rho u) = 0, \\ (\rho u)_t + \nabla \cdot (\rho u \otimes u + T^i \rho \mathbb{I}) = \rho \nabla \phi, \\ e^{-\phi} = \rho. \end{cases} \tag{1.4}$$

Rewriting the above equation, we get the following isothermal Euler equation

$$\begin{cases} \rho_t + \nabla \cdot (\rho u) = 0, \\ u_t + u \cdot \nabla u + (T^i + 1) \nabla \ln \rho = 0. \end{cases} \tag{1.5}$$