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Existence of solution for a nonvariational elliptic system with exponential growth in dimension two

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Abstract

We prove existence of solution for an elliptic system on a bounded domain in dimension two. We use the Galerkin scheme in the product of Hilbert spaces. The nonlinearities may have subcritical or critical exponential growth.

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Keywords: Elliptic system; Existence of solution; Subcritical and critical exponential growth; Galerkin scheme

1. Introduction

We prove existence of solution of the system

$$\begin{cases} -\Delta v = \lambda v^{q_1} + f(u) & \text{in} \quad \Omega \\ -\Delta u = \sigma u^{q_2} + g(v) & \text{in} \quad \Omega \\ v, u > 0 & \text{in} \quad \Omega \\ v = u = 0 & \text{on} \quad \partial\Omega, \end{cases} \tag{1}$$

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where $\Omega \subset \mathbb{R}^2$ is a bounded domain with smooth boundary, $\lambda, \sigma > 0$ are parameters, $0 < q_1, q_2 < 1$, $f, g : \mathbb{R} \to \mathbb{R}$ is a continuous functions and

$$0 \le f(s)s \le C|s|^{p_1} \exp(\alpha s^2), \tag{2}$$

$$0 \le g(s)s \le C|s|^{p_2} \exp(\beta s^2) \tag{3}$$

where $2 < p_1, p_2 < \infty$, $\alpha, \beta > 0$ and $C > 0$ are constants.

Remark 1.1. The results of this paper also work for a system with $u_1, ..., u_m$ variables and m equations $-\Delta u_i = \lambda_i u_i^{q_i} + f_i(u_j)$ where $j = \sigma(i)$, $\sigma : \{1, 2, ..., m\} \to \{1, 2, ..., m\}$ is a permutation such that $\sigma^k(i) \neq i$ for $k = 1, 2, ..., m-1$ and $\sigma^m(i) = i$, the index k stands for composition of functions, see [1] for the concept of m-coupled elliptic systems. Conditions (2)–(3) should be changed accordingly for each f_i.

We state our main result.

Theorem 1.1. *Suppose that $f, g : \mathbb{R} \to \mathbb{R}$ are continuous functions satisfying (2) and (3) respectively. Then there exist $\lambda^*, \sigma^* > 0$ such that for every $\lambda \in (0, \lambda^*)$ and $\sigma \in (0, \sigma^*)$ the problem (1) has positive weak solutions $v, u \in H_0^1(\Omega) \cap H^2(\Omega)$.*

A function h has subcritical growth at ∞ if for every $\gamma > 0$

$$\lim_{s\to\infty} \frac{|h(x,s)|}{e^{\gamma s^2}} = 0$$

The critical growth of h at ∞ means that there is $\gamma_0 > 0$ such that

$$\lim_{s\to\infty} \frac{|h(x,s)|}{e^{\gamma s^2}} = 0 \quad \forall \gamma > \gamma_0 \quad \text{and} \quad \lim_{s\to\infty} \frac{|h(x,s)|}{e^{\gamma s^2}} = \infty \quad \forall \gamma < \gamma_0.$$

Equations like $-\Delta u = h(x, u)$ with h having critical or subcritical growth have been studied in [2–7]. The Trudinger–Moser inequality [8–10] has a crucial role, since it indicates the space of functions one has to work to seek for solutions. The inequality reads as follows. Given $u \in H_0^1(\Omega)$, then

$$e^{\zeta |u|^2} \in L^1(\Omega) \ \text{for every} \ \zeta > 0, \tag{4}$$

and there exists a positive constant L such that

$$\sup_{\|u\|_{H_0^1(\Omega)} \le 1} \int_\Omega e^{\xi |u|^2} dx \le L \ \text{for every} \ \xi \le 4\pi. \tag{5}$$

Elliptic systems of type

$$\begin{cases} -\Delta v = f(x, v, u) & \text{in} \quad \Omega \\ -\Delta u = g(x, v, u) & \text{in} \quad \Omega \\ v = u = 0 & \text{on} \quad \partial\Omega, \end{cases} \tag{6}$$

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