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Global well-posedness and decay estimates of strong solutions to a two-phase model with magnetic field

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Abstract

In this paper, we consider the Cauchy problem for a two-phase model with magnetic field in three dimensions. The global existence and uniqueness of strong solution as well as the time decay estimates in $H^2(\mathbb{R}^3)$ are obtained by introducing a new linearized system with respect to $(n^\gamma - \tilde{n}^\gamma, n - \tilde{n}, P - \tilde{P}, u, H)$ for constants $\tilde{n} \geq 0$ and $\tilde{P} > 0$, and doing some new a priori estimates in Sobolev Spaces to get the uniform upper bound of $(n - \tilde{n}, n^\gamma - \tilde{n}^\gamma)$ in $H^2(\mathbb{R}^3)$ norm.

MSC: 76T10; 76D03; 74H40; 35Q83; 76Wxx

Keywords: Two-phase model; Drift-flux model; Vlasov–Fokker–Planck/magnetohydrodynamics equations; Global well-posedness; Decay estimates

Contents

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1. Introduction

In the paper, we consider a two-phase system with magnetic field modelling the motion of the mixture of fluid and particles. Mathematically, the model is stated as below.

$$\begin{cases} n_t + \mathrm{div}(nu) = 0, \\ \rho_t + \mathrm{div}(\rho u) = 0, \\ [(\rho+n)u]_t + \mathrm{div}((\rho+n)u \otimes u) + \nabla P - \mu\triangle u - (\lambda+\mu)\nabla \mathrm{div} u = (\nabla \times H) \times H, \\ H_t - \nabla \times (u \times H) = -\nabla \times (\nu \nabla \times H), \\ \mathrm{div} H = 0, \end{cases} \tag{1.1}$$

with $t \geq 0$ and $x = (x_1, x_2, x_3) \in \mathbb{R}^3$. The viscosity coefficients and pressure satisfy

$$\mu > 0, \ \lambda + \frac{2}{3}\mu \geq 0 \tag{1.2}$$

and

$$P = \rho + An^\gamma \tag{1.3}$$

for some constants $A > 0$ and $\gamma > 1$, respectively. Here the density of the fluid $n \geq 0$, the fluid velocity field $u = (u^1, u^2, u^3)$, and the density of the particles in the mixture $\rho \geq 0$ is related to the probability distribution function $F(t, x, \xi)$ in the macroscopic description through the relation

$$\rho(x,t) = \int_{\mathbb{R}^3} F(x,t,\xi)d\xi.$$

H represents the magnetic field. The constant $\nu > 0$ is the resistivity coefficient.

Without the viscosity terms and magnetic field in (1.1)–(1.3), this system was first derived by Carrillo and Goudon [3] by virtue of taking scaling limits from the Vlasov–Fokker–Planck/compressible Euler equations. They introduced the flowing regime and the bubbling regime with respect to two different scalings and investigated the stability and asymptotic limits. The system (1.1)–(1.3) is known as the case of the flow regime. From mathematical points of view, the main differences between the cases of the flowing regime and the bubbling regime are that the density of the particles solves a parabolic equation for the latter case. For the case of bubbling regime, when the viscous effects are taken into account, there are some previous works on it. For the