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The L^p Robin problem for Laplace equations in Lipschitz and (semi-)convex domains ☆

Sibei Yang [a], Dachun Yang [b,*], Wen Yuan [b]

[a] *School of Mathematics and Statistics, Gansu Key Laboratory of Applied Mathematics and Complex Systems, Lanzhou University, Lanzhou 730000, People's Republic of China*

[b] *School of Mathematical Sciences, Beijing Normal University, Laboratory of Mathematics and Complex Systems, Ministry of Education, Beijing 100875, People's Republic of China*

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Abstract

Let $n \geq 3$ and Ω be a bounded Lipschitz domain in $\mathbb{R}^n$. Assume that $p \in (2, \infty)$ and the function $b \in L^\infty(\partial\Omega)$ is non-negative, where $\partial\Omega$ denotes the boundary of Ω. Denote by ν the outward unit normal to $\partial\Omega$. In this article, the authors give two necessary and sufficient conditions for the unique solvability of the Robin problem for the Laplace equation $\Delta u = 0$ in Ω with boundary data $\partial u/\partial\nu + bu = f \in L^p(\partial\Omega)$, respectively, in terms of a weak reverse Hölder inequality with exponent p or the unique solvability of the Robin problem with boundary data in some weighted $L^2(\partial\Omega)$ space. As applications, the authors obtain the unique solvability of the Robin problem for the Laplace equation in the bounded (semi-)convex domain Ω with boundary data in (weighted) $L^p(\partial\Omega)$ for any given $p \in (1, \infty)$.

MSC: primary 35J25; secondary 35J05, 42B25

Keywords: (Semi-)convex domain; Lipschitz domain; Laplace equation; Robin problem; Weak reverse Hölder inequality

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* Corresponding author.
E-mail addresses: yangsb@lzu.edu.cn (S. Yang), dcyang@bnu.edu.cn (D. Yang), wenyuan@bnu.edu.cn (W. Yuan).

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Contents

1. Introduction

Let Ω be a bounded Lipschitz domain in $\mathbb{R}^n$ with $n \geq 3$. Denote by $\nu := (\nu_1, \ldots, \nu_n)$ the *outward unit normal* to $\partial\Omega$, the boundary of Ω. Moreover, for a measurable function F on Ω, the *non-tangential maximal function* of F is defined by setting, for any $P \in \partial\Omega$,

$$(F)^*(P) := \sup\left\{|F(x)| : x \in \Omega, \ |x - P| < \widetilde{C}\delta(x)\right\}, \tag{1.1}$$

where $\delta(x) := \mathrm{dist}(x, \partial\Omega) := \inf\{|x - z| : z \in \partial\Omega\}$ and $\widetilde{C} \in (1, \infty)$ is a positive constant, which may depend on Ω, but is independent of $P \in \partial\Omega$ and $x \in \Omega$.

Now we recall the definition of $A_p(\partial\Omega)$ weights (see, for example, [29, p. 4] and [28, (7.1)]).

Definition 1.1. Let $\Omega \subset \mathbb{R}^n$ be a domain and $p \in [1, \infty)$. A non-negative and locally integrable function ω on $\partial\Omega$ is called an $A_p(\partial\Omega)$ *weight* if there exists a positive constant C such that, for any $Q \in \partial\Omega$ and $r \in (0, \mathrm{diam}(\partial\Omega))$,

$$\left\{\frac{1}{\sigma(I(Q,r))}\int_{I(Q,r)} w(x)\, d\sigma(x)\right\}\left\{\frac{1}{\sigma(I(Q,r))}\int_{I(Q,r)} [w(x)]^{-\frac{1}{p-1}}\, d\sigma(x)\right\}^{p-1} \leq C \tag{1.2}$$

when $p \in (1, \infty)$, and

$$\frac{1}{\sigma(I(Q,r))}\int_{I(Q,r)} w(x)\, d\sigma(x) \leq C\left\{\operatorname*{ess\,inf}_{x\in I(Q,r)} w(x)\right\} \tag{1.3}$$

when $p = 1$, where $\mathrm{diam}(\partial\Omega) := \sup\{|x - y| : x, y \in \partial\Omega\}$, $I(Q,r) := B(Q,r) \cap \partial\Omega$ and $d\sigma$ denotes the surface measure on $\partial\Omega$. The smallest constant C as above such that (1.2) or (1.3) holds is called the $A_p(\partial\Omega)$ *weight constant* of ω and denoted by $[\omega]_{A_p(\partial\Omega)}$. Here and hereafter,

$$B(Q,r) := \{x \in \mathbb{R}^n : |x - Q| < r\}.$$

Let $p \in (1, \infty)$ and $\omega \in A_q(\partial\Omega)$ for some $q \in [1, \infty)$. Recall that the weighted space $L^p_\omega(\partial\Omega)$ is defined by setting

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