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Energy equality for the 3D critical convective Brinkman–Forchheimer equations

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Abstract

In this paper we give a simple proof of the existence of global-in-time smooth solutions for the convective Brinkman–Forchheimer equations (also called in the literature the tamed Navier–Stokes equations)

$$\partial_t u - \mu \Delta u + (u \cdot \nabla) u + \nabla p + \alpha u + \beta \left| u \right|^{r-1} u = 0$$

on a 3D periodic domain, for values of the absorption exponent r larger than 3. Furthermore, we prove that global, regular solutions exist also for the critical value of exponent $r = 3$, provided that the coefficients satisfy the relation $4\mu\beta \geq 1$. Additionally, we show that in the critical case every weak solution verifies the energy equality and hence is continuous into the phase space L^2. As an application of this result we prove the existence of a strong global attractor, using the theory of evolutionary systems developed by Cheskidov.
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1. Introduction

In this paper we consider both weak and strong solutions of the three-dimensional incompressible convective Brinkman–Forchheimer equations (CBF)

$$\begin{cases} \partial_t u - \mu \Delta u + (u \cdot \nabla) u + \nabla p + \alpha u + \beta |u|^{r-1} u = 0, \\ \operatorname{div} u = 0, \\ u(x,0) = u_0(x), \end{cases} \tag{1.1}$$

where $u(x,t) = (u_1, u_2, u_3)$ is the velocity field and the scalar function $p(x,t)$ is the pressure. The constant μ denotes the positive Brinkman coefficient (effective viscosity). The positive constants α and β denote respectively the Darcy (permeability of porous medium) and Forchheimer (proportional to the porosity of the material) coefficients. The exponent r can be greater than or equal to 1. The domain on which we consider problem (1.1) is a three-dimensional torus $\mathbb{T}^3 = [0, 2\pi]^3$, with periodic boundary conditions and zero mean-value constraint for the functions, i.e. $\int_{\mathbb{T}^3} u(x,t)\,\mathrm{d}x = 0$. Very similar arguments should work to prove most of the results given here (except perhaps the energy equality result) also for the Cauchy problem, i.e. in the case when the domain is the whole space $\mathbb{R}^3$. However, when $\Omega \subset \mathbb{R}^3$ is an open, bounded domain with Dirichlet boundary conditions $u|_{\partial\Omega} = 0$, not all the results proved here are that straightforward. In particular, one has to be very careful in choosing an approximation in the proof of Theorem 1.4, and we will address this problem in a future paper.

The CBF equations (1.1) describe the motion of incompressible fluid flows in a saturated porous medium. While the motivation for introducing an absorption term $|u|^{r-1} u$ is purely mathematical, this model is used in connection with some real world phenomena, e.g. in the theory of non-Newtonian fluids as well as tidal dynamics (see [7], [17], [22] and references therein). However, its applicability is believed to be limited to flows when the velocities are sufficiently high and porosities are not too small, i.e. when the Darcy law for a porous medium no longer applies (for more details see [13], [12] and the discussion in [11]).

In this paper we use the standard notation for the vector-valued function spaces which often appear in the theory of fluid dynamics. For an arbitrary domain $\Omega \subseteq \mathbb{R}^n$ we define:

$$\begin{aligned} C_0^\infty(\Omega) &:= \left\{ \varphi \in C^\infty(\Omega) : \operatorname{supp} \varphi \text{ is compact} \right\}, \\ \mathcal{D}_\sigma(\Omega) &:= \left\{ \varphi \in C_0^\infty(\Omega) : \operatorname{div} \varphi = 0 \right\}, \\ X_q(\Omega) &:= \text{closure of } \mathcal{D}_\sigma(\Omega) \text{ in the Lebesgue space } L^q(\Omega), \\ V^s(\Omega) &:= \text{closure of } \mathcal{D}_\sigma(\Omega) \text{ in the Sobolev space } W^{s,2}(\Omega) \quad \text{for} \quad s > 0. \end{aligned}$$

The space of divergence-free test functions in the space–time domain is denoted by

$$\mathcal{D}_\sigma(\Omega_T) := \left\{ \varphi \in C_0^\infty(\Omega_T) : \operatorname{div} \varphi(\cdot, t) = 0 \right\},$$

where $\Omega_T := \Omega \times [0, T)$ for $T > 0$. Note that $\varphi(x, T) = 0$ for all $\varphi \in \mathcal{D}_\sigma(\Omega_T)$.

We denote the Hilbert space $X_2(\Omega)$ by H, $V^1(\Omega)$ by V and $V^s(\Omega)$ by V^s for $s \neq 1$. The space H is endowed with the inner product induced by $L^2(\Omega)$. We denote it by $\langle \cdot, \cdot \rangle$, and the corresponding norm is denoted by $\|\cdot\|$.

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