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Study on monostable and bistable reaction–diffusion equations by iteration of travelling wave maps

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Abstract

In this paper, based on the iterative properties of travelling wave maps, we develop a new method to obtain spreading speeds and asymptotic propagation for monostable and bistable reaction–diffusion equations. Precisely, for Dirichlet problems of monostable reaction–diffusion equations on the half line, by making links between travelling wave maps and integral operators associated with the Dirichlet diffusion kernel (the latter is NOT invariant under translation), we obtain some iteration properties of the Dirichlet diffusion and some a *priori* estimates on nontrivial solutions of Dirichlet problems under travelling wave transformation. We then provide the asymptotic behavior of nontrivial solutions in the space–time region for Dirichlet problems. These enable us to develop a unified method to obtain results on heterogeneous steady states, travelling waves, spreading speeds, and asymptotic spreading behavior for Dirichlet problem of monostable reaction–diffusion equations on $\mathbb{R}_+$ as well as of monostable/bistable reaction–diffusion equations on $\mathbb{R}$.

Keywords: Dirichlet problem; Heterogeneous steady state; Monostable/bistable reaction–diffusion equation; Spreading speed; Travelling wave map

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1. Introduction

Consider the following reaction–diffusion equation,

$$\frac{\partial u}{\partial t}(t,x) = \Delta u(t,x) + f(u(t,x)), \tag{1.1}$$

where Δ is the Laplacian operator on a domain Ω in $\mathbb{R}^N$ and $f : \mathbb{R} \to \mathbb{R}$ is a continuously differentiable function. When Ω is a bounded domain in $\mathbb{R}^N$, various boundary conditions can be posed, among which are the typical Dirichlet boundary value condition and Neumann boundary value condition. By using the energy function and standard parabolic regularity estimates, one can easily conclude that each bounded solution approaches the set of equilibria of (1.1) [5,23]. When Ω is unbounded, say $\mathbb{R}^N$, a standard energy functional that serves as a Lyapunov function on bounded domains may not even be well defined. By applying the zero number technique and dynamical system methods involving invariant manifolds in neighborhoods of equilibria, the convergence for several types of equations and a large class of initial functions has been established (see [7–10,15] and the references therein).

For an unbounded spatial domain, say $\mathbb{R}^N$, travelling wave solutions and asymptotic propagation are two important topics [1,13,18]. Travelling wave solutions may quite often determine the long term behavior of other solutions while asymptotic propagation describes how other solutions converge to an equilibrium as $t \to \infty$. The study on travelling waves can be traced back to the celebrated papers of Fisher [14] and Kolmogorov et al. [18] while the asymptotic spreading speeds of (1.1) have been established mathematically by Aronson and Weinberger [1]. Since solutions of initial value problems of reaction–diffusion equations can be considered as solutions to some discrete dynamical systems in appropriate spaces, Weinberger [25] and Lui [21] established the theory of spreading speeds for monotone *discrete* dynamical systems. This theory has been further developed recently in [11,12,16,19,20,26,28,30] for more general *monotone/non-monotone* semiflows so that it can be applied to a variety of discrete and continuous time evolution equations in homogeneous or periodic media. By using the Harnack inequality up to boundary and the maximum principle as well as the strict positivity of solutions instead of dynamical system methods, Berestycki et al. [3,4] have outlined a different theory of various asymptotic spreading speeds for monostable problems under general heterogeneous periodic framework or for (1.1) in non-periodic spacial domains under Neumann boundary conditions. In contrast to Weinberger [1,25,26], almost all results in Berestycki et al. [4] were established without assuming periodicity of spacial domains.

However, for the Dirichlet problem in non-periodic domains including the half line, none of the above mentioned theories is applicable. On the one hand, the solution mappings of such Dirichlet problems are not translation invariant. But, one of the basic assumptions of the theory of Weinberger et al. is that the monotone semiflow is invariant under translation. In fact, their argument and representation processes depend very heavily on this translation invariance hypothesis. Therefore, the theory and method of Weinberger et al. cannot be applied directly to such Dirichlet problems. We should emphasize that, even for some very special non-translation invariant systems, how to link them with some translation invariant systems and then to get asymptotic propagation characteristics is not trivial as the choices of the translation invariant systems associated with the solution mappings as well as links between the iterations are not an easy and natural thing. On the other hand, because the Harnack inequality cannot be extended to the boundary for Dirichlet problems and solutions for Dirichlet problems are zeros on the boundary, the methods

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