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Half space problem for Euler equations with damping in 3-D

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Abstract

In this paper, we consider the half space problem for Euler equations with damping in 3-D. We restudy the fundamental solution for the Cauchy problem to obtain an exponentially sharp pointwise structure and a clear decomposition of the singular-regular components. Later, both Green's function for initial boundary value problem and fundamental solutions for Cauchy problems are investigated in the transformed domain after Laplace transform. The symbols are obtained and a connection between Green's function and fundamental solutions are established for the pointwise space–time structure of Green's function. Finally, the sharp estimates for Green's function together with a priori estimates from the energy method for high order derivatives result in the nonlinear stability of the solution and also the decaying rates.

Keywords: Euler equations; Half space problem; Green's function; Symbols in transformed domain; Pointwise structure

1. Introduction

The 3-dimensional Euler equations with damping

$$\begin{cases} \rho_\tau + div(\rho\vec{u}) = 0, \\ (\rho u^j)_\tau + div(\rho\vec{u}u^j) + \rho^\gamma_{\eta_j} = -\kappa\rho u^j, \quad \vec{u} = (u^1, u^2, u^3) \in \mathbb{R}^3 \end{cases} \tag{1.1}$$

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model the fluid in a porous media. Here, $\kappa > 0$ is a constant to model the magnitude of the viscosity and ρ^γ, $\gamma \geq 1$, is the pressure for an isentropic gas flow with a given γ-law. In this paper, we consider (1.1) in the half space $\eta_1 > b\tau$ with a negative and subsonic boundary speed b

$$-\sqrt{\gamma} < b < 0, \tag{1.2}$$

and under the following initial and boundary conditions:

$$(\rho, \vec{u})\big|_{\tau=0} = (1 + \sigma_0, \vec{u}_0)(\vec{\eta}), \tag{1.3}$$

$$\begin{cases} \sqrt{\gamma}(\rho - 1) + u^1\big|_{\eta_1 = b\tau} = 0, \\ u^2\big|_{\eta_1 = b\tau} = 0, \\ u^3\big|_{\eta_1 = b\tau} = 0. \end{cases} \tag{1.4}$$

The time-asymptotic behavior for Cauchy problem of the system (1.1) in 1-D case was initiated in Hsiao–Liu [9]. Then the Cauchy problems in both 1-D case and multi-D case have been widely studied. One can refer to [6,7,15–18,27,28,30,35–40] and the references there.

For the half space problem in 1-D, the Dirichlet boundary condition case and Neumann boundary condition case are studied in [23–25,31]. Nishihara–Yang [31] investigated the half space problem with Dirichlet boundary condition and Neumann boundary condition respectively. In Marcati–Mei [24], another kind of Dirichlet boundary condition is posed. In the above cases, the solutions are proved to tend to the diffusion wave and the time convergence rates are obtained. Later, Marcati–Mei–Rubino improved the convergence rates in [25]. In Ma–Mei [23], better asymptotic profiles were given. In Deng [2], the mixed type boundary condition was considered and with constructed Green's function and a priori estimates from energy method, the pointwise structure of the solution was obtained.

Few results are about the multi-D case with boundary. In Liu–Wang [22], the authors used the energy method to prove the existence and stability of the solutions for half space problem in 2-D with a moving boundary $x_1 = bt$ and assuming that the speed of the boundary is positive and subsonic:

$$0 < b < c \ (c \text{ is the sound speed}).$$

Under the same assumption on the boundary speed, the exponentially convergence rate in both time and space variables was obtained by weighted energy method in [1].

For the problem in a bounded domain, one can refer to [11,33,34] and the references there. For the nonisentropic case with or without a boundary, one can refer to [7,8,10,12–14,26,29,32] and the references there.

In this paper, we are interested with the pointwise space time structure of the solutions for half space problem (1.1)–(1.4). Compared with [1], the negative boundary speed assumption (1.2) makes it far more difficult to obtain the convergence rate by simply weighted energy method since the rate for this problem is algebraic. Besides, we can hardly use the previous Green's function method for 1-D problem initiated in [19], since the singularity propagation obtained from Green's identity may even be not well defined for a multi-D half space problem. Here, we adopt a more generalized Green's function method initiated in [20] and also make improvement of the procedure.