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Journal of Differential Equations

www.elsevier.com/locate/jde

On circular flows: Linear stability and damping

Christian Zillinger

Mathematisches Institut, Universität Bonn, 53115 Bonn, Germany

Received 21 May 2016; revised 25 May 2017

Abstract

In this article we establish linear inviscid damping with optimal decay rates around 2D Taylor–Couette flow and similar monotone flows in an annular domain $B_{r_2}(0) \setminus B_{r_1}(0) \subset \mathbb{R}^2$. Following recent results by Wei, Zhang and Zhao [10], we establish stability in weighted norms, which allow for a singularity formation at the boundary, and additionally provide a description of the blow-up behavior.

MSC: 35Q31; 35Q36; 76E05; 76B03

Keywords: 2D Euler equations; Linear inviscid damping; Taylor–Couette flow; Boundary effects; Boundary layer; Blow-up

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E-mail address: zill@math.uni-bonn.de.

http://dx.doi.org/10.1016/j.jde.2017.08.026
0022-0396/

Please cite this article in press as: C. Zillinger, On circular flows: Linear stability and damping, J. Differential Equations (2017), http://dx.doi.org/10.1016/j.jde.2017.08.026

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1. Introduction

In this article we consider the linear stability and long-time asymptotic behavior of circular flows in an annular domain $(x, y) \in B_{r_2}(0) \setminus B_{r_1}(0)$. Such two-dimensional flows can for example be established experimentally in rotating cylinders, where the rotation is sufficiently slow as to not cause a (three-dimensional) Taylor–Couette instability.

In this setting, radial vorticities

$$\begin{aligned} \omega(x, y) &= \omega(r), \\ v(x, y) &= \partial_r \psi e_\theta = \begin{pmatrix} -y \\ x \end{pmatrix} \frac{\psi'(r)}{r}, \\ \psi''(r) + \frac{1}{r}\psi'(r) &= \omega(r), \end{aligned} \tag{1}$$

are stationary solutions of the incompressible 2D Euler equations.

Considering a small perturbation to Taylor–Couette flow,

$$\frac{\psi'(r)}{r} = A + \frac{B}{r^2}, \tag{2}$$

we observe in Fig. 1 that for $B = 0$, i.e. constant angular velocity, perturbations are rotated while keeping their shape. However, in the general case when $B \neq 0$, $\frac{\psi'(r)}{r}$ is strictly monotone and the perturbation is sheared in way reminiscent of plane Couette flow, as is depicted in Fig. 2. This mixing behavior underlies the phenomenon of (linear) inviscid damping.

Considering polar coordinates, the linearized Euler equations around these stationary solutions are given by

$$\begin{aligned} \partial_t f + U(r)\partial_\theta f &= b(r)\partial_\theta \phi, \\ (\partial_r^2 + \frac{1}{r}\partial_r + \frac{1}{r^2}\partial_\theta^2)\phi &= f, \\ \partial_\theta \phi|_{r=r_1,r_2} &= 0, \\ (t, \theta, r) &\in \mathbb{R} \times \mathbb{T} \times [r_1, r_2], \end{aligned} \tag{3}$$

where U and b are given by

$$\begin{aligned} U(r) &= \frac{\psi'(r)}{r}, \\ b(r) &= -\frac{1}{r}\partial_r(\partial_r^2 \psi(r) + \frac{1}{r}\partial_r \psi(r)), \end{aligned}$$

and $b(r) \equiv 0$ if and only if one considers Taylor–Couette flow, $U(r) = A + \frac{B}{r^2}$.

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