Accepted Manuscript

Recovering a purely atomic finite measure from its range

Artur Bartoszewicz, Szymon Głąb, Jacek Marchwicki

 PII:
 S0022-247X(18)30608-5

 DOI:
 https://doi.org/10.1016/j.jmaa.2018.07.026

 Reference:
 YJMAA 22417

To appear in: Journal of Mathematical Analysis and Applications

Received date: 7 May 2018

<page-header><text><section-header><text><section-header>

Please cite this article in press as: A. Bartoszewicz et al., Recovering a purely atomic finite measure from its range, *J. Math. Anal. Appl.* (2018), https://doi.org/10.1016/j.jmaa.2018.07.026

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

ACCEPTED MANUSCRIPT

RECOVERING A PURELY ATOMIC FINITE MEASURE FROM ITS RANGE

ARTUR BARTOSZEWICZ, SZYMON GŁĄB, AND JACEK MARCHWICKI

ABSTRACT. Let μ be a purely atomic finite measure. By the range of μ we understand the set $\operatorname{rng}(\mu) = \{\mu(E) : E \subset \mathbb{N}\}$. We are interested in the two following questions. Which set can be a range of some measure μ ? Can the purely atomic measure μ be uniquely recovered from its range?

1. INTRODUCTION

Assume that μ is a purely atomic finite measure. We may assume that μ is defined on \mathbb{N} and $\mu(\{n\}) \ge \mu(\{n+1\})$. Throughout the paper we assume that measures are always purely atomic, finite and they are defined on \mathbb{N} such that their n+1-st atoms have measures not greater than their n-th atoms. We are interested in the following questions:

- For which subsets R of \mathbb{R} there is a measure μ such that R is its range (i.e. $R = \operatorname{rng}(\mu) := \{\mu(E) : E \subset \mathbb{N}\}$)?
- For which subsets R of \mathbb{R} there is exactly one measure μ with $R = \operatorname{rng}(\mu)$?

To simplify the notation let $x_n = \mu(\{n\})$ be a measure of the *n*-th largest atom of μ . Note that

$$\operatorname{rng}(\mu) = \{\mu(E) : E \subset \mathbb{N}\} = \{\sum_{n \in E} \mu(\{n\}) : E \subset \mathbb{N}\} = \{\sum_{n=1}^{\infty} \varepsilon_n x_n : \varepsilon_n = \{0, 1\}^{\mathbb{N}}\}.$$

The latter set is also denoted by $A(x_n)$ and it is called the achievement set of (x_n) (see [16]). Let us present here two simple examples.

Example 1.1. Consider the procedure of rolling dice until the value on the dice is less than 5. For $E \subset \mathbb{N}$ let $\mu_1(E)$ be the probability that the procedure stops for some n from E. Then $\mu_1(\{n\}) = \frac{2}{3^n}$. It is easy to see that for $x_n = \mu_1(\{n\})$ the set $A(x_n)$, or $\operatorname{rng}(\mu_1)$, is equal to the classical Cantor ternary set C.

Example 1.2. Consider the procedure of tossing a fair coin until the head appears. For $E \subset \mathbb{N}$ let $\mu_2(E)$ be the probability that the procedure stops for some n from E. Then $\mu_2(\{n\}) = \frac{1}{2^n}$ and $\operatorname{rng}(\mu_2) = [0, 1]$.

Achievement sets of sequences, defined for all summable sequences (x_n) , have been considered by many authors; some results have been rediscovered several times. Let us list basic properties of $A(x_n)$ (some of them were observed by Kakeya in [17] in 1914):

- (i) $A(x_n)$ is a compact perfect or finite set,
- (ii) If $|x_n| > \sum_{i>n} |x_i|$ for all sufficiently large n's, then $A(x_n)$ is homeomorphic to the ternary Cantor set C,
- (iii) If $|x_n| \leq \sum_{i>n} |x_i|$ for all sufficiently large *n*'s, then $A(x_n)$ is a finite union of closed intervals. Moreover, if $|x_n| \geq |x_{n+1}|$ for all but finitely many *n*'s and $A(x_n)$ is a finite union of closed intervals, then $|x_n| \leq \sum_{i>n} |x_i|$ for all but finitely many *n*'s.

²⁰¹⁰ Mathematics Subject Classification. Primary: 40A05 ; Secondary: 11K31.

Key words and phrases. iterated function system, purely atomic measure, achievement set, set of subsums, absolutely convergent series.

Download English Version:

https://daneshyari.com/en/article/8899169

Download Persian Version:

https://daneshyari.com/article/8899169

Daneshyari.com