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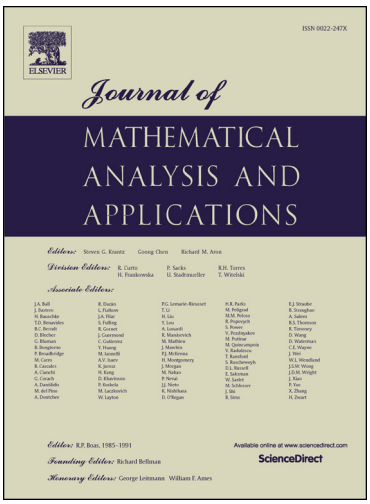


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# Conditions on Unimodality and Logconcavity for Densities of Coherent Systems with an Application to Bernstein Operators

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**Abstract**

In this note, the distribution of the lifetime of a coherent system with independent and identically distributed component lifetimes is considered. Conditions yielding unimodality or logconcavity of the density function of the system lifetime are obtained. In the conditions, only assumptions on the density function of the components and on the signature of the system are imposed. The results are illustrated with several examples. Additionally, a problem on preservation of logconcavity under the classical Bernstein operator is solved.

Keywords: unimodality, logconcavity, signature, coherent system, variation diminishing property, Bernstein polynomial

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## 1 Introduction

In the setting of coherent systems with independent and identically distributed (iid) component lifetimes, conditions for unimodality and logconcavity of the density function of the system lifetime distribution have been studied. Alam (1972) has obtained conditions for unimodality of the lifetime distribution of $k$-out-of-$n$ systems. An extension to coherent system is given in Sabnis and Nair (1997). Recently, Bieniek and Burkschat (2018) derived conditions on the signature of a coherent system which yield unimodality or bimodality in the case of uniformly distributed component lifetimes. Logconcavity of the density of the system lifetime has been examined in Barlow and Proschan (1966), Huang and Ghosh (1982) and Franco et al. (2003). Moreover, further results on logconcavity can be found, e.g., in Barlow and Proschan (1981), Finner and Roters (1993), An (1998), Sengupta and Nanda (1999), Bagnioli and Bergstrom (2005), Marshall and Olkin (2007) and Alimohammadi et al. (2016). Unimodality and related properties are also central topics of the monograph by Dharmadhikari and Joag-dev (1988). Recently, the problem of preservation of logconcavity under the classical Bernstein operator has been studied by Badía (2009) and Badía and Sangüesa (2014).

It is well-known that the distribution function of the system lifetime $T$ of a coherent system consisting of $n$ iid components with underlying continuous distribution function $F$ possesses the

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