J. Math. Anal. Appl. ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Existence and concentrating behavior of solutions for Kirchhoff type equations with steep potential well ☆

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ARTICLE INFO

Article history:
Received 15 January 2018
Available online xxxx
Submitted by M. Musso

Keywords:
Kirchhoff type equation
Asymptotically linear
Ground state solution
Variational methods

ABSTRACT

In this paper, we consider the following Kirchhoff type equations

$$\begin{cases} -\Big(a+b\int\limits_{\mathbb{R}^3}|\nabla u|^2\Big)\Delta u+\lambda V(x)u=q(x)f(u) \text{ in } \mathbb{R}^3, \\ u\in H^1(\mathbb{R}^3), \end{cases} \tag{0.1}$$

where a, b, $\lambda > 0$, $V \in C(\mathbb{R}^3, \mathbb{R})$ is a potential well, $q(x)$ is a positive bounded function, $f(s)$ is either asymptotically linear or asymptotically 3-linear in s at infinity. Under some other suitable conditions on V, q and f, the existence, nonexistence and concentrating behavior of solutions to problem (0.1) are obtained by using variational methods. We mainly extend the results in J. Sun and T. Wu (2014) [26], which dealt with Kirchhoff type equations with positive potential well, to Kirchhoff type equations with sign-changing potential well.
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1. Introduction and main result

In this paper, we consider the following Kirchhoff type equations

$$\begin{cases} -\Big(a+b\int\limits_{\mathbb{R}^3}|\nabla u|^2\Big)\Delta u+\lambda V(x)u=q(x)f(u) \text{ in } \mathbb{R}^3, \\ u\in H^1(\mathbb{R}^3) \end{cases} \tag{1.1}$$

where a, b, $\lambda > 0$, $q(x)$ is a positive bounded function, $f(s)$ is either asymptotically linear or asymptotically 3-linear in s at infinity, and the potential V satisfies the following conditions:

☆ This work is supported by the National Natural Science Foundation of China under Grant No. 11771166.
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https://doi.org/10.1016/j.jmaa.2018.07.052
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Please cite this article in press as: H. Jia, X. Luo, Existence and concentrating behavior of solutions for Kirchhoff type equations with steep potential well, J. Math. Anal. Appl. (2018), https://doi.org/10.1016/j.jmaa.2018.07.052

(V_1) $V \in C(\mathbb{R}^3, \mathbb{R})$ and V is bounded from below;
(V_2) there exists $z > 0$ such that the set $\{x \in \mathbb{R}^3 : V(x) < z\}$ is nonempty and has finite measure;
(V_3) $\Omega = intV^{-1}(0)$ is nonempty and has smooth boundary with $\overline{\Omega} = V^{-1}(0)$.

This kind of hypotheses on V was first introduced by Bartsch et al. in [3,5] to study the nonlinear Schrödinger equation. In particular, the usually concerned harmonic trapping potential $V(x) = \omega_1|x_1|^2 + \omega_2|x_2|^2 + \omega_3|x_3|^2 - \omega$ with $\omega, \omega_i > 0$ $(i = 1, 2, 3)$ satisfies (V_1)–(V_3), where ω_i is called the anisotropy factor of the trap in quantum physics and trapping frequency of the ith-direction in mathematics, see [6,7,22,23].

λV represents a potential well whose depth is controlled by λ and is called the steep potential well if λ is sufficiently large. In recent years, elliptic equations with steep potential well received much attention of researchers, see [3,4,25–27,31].

Problem (1.1) arises in a famous physical context [16]. Indeed, if we set $V(x) = 0$ and replace $\mathbb{R}^3$ by a bounded domain $\Omega_0 \subset \mathbb{R}^3$ in (1.1), then we get the following Kirchhoff Dirichlet problem

$$\begin{cases} -\Big(a + b\displaystyle\int_{\Omega_0} |\nabla u|^2\Big)\Delta u = f(x, u), & x \in \Omega_0, \\ u = 0, & x \in \partial\Omega_0. \end{cases} \tag{1.2}$$

In recent years, a great interests have been devoted to problem (1.2), which is related to the stationary analogue of the equation

$$\rho \frac{\partial^2 u}{\partial t^2} - \Big(\frac{P_0}{h} + \frac{E}{2L}\int_0^L |\frac{\partial u}{\partial x}|^2\Big)\frac{\partial^2 u}{\partial x^2} = 0 \tag{1.3}$$

presented by Kirchhoff in [16] as an existence of the classical D'Alembert's wave equations for free vibration of elastic strings. In [19], J. Lions introduced an abstract functional analysis framework to the following equation

$$u_{tt} - \Big(a + b\int_{\Omega} |\nabla u|^2\Big)\triangle u = f(x, u). \tag{1.4}$$

After that, (1.4) received much attention, see [1,2,8,9] and the references therein.

Kirchhoff's model takes into account the changes in length of the string produced by transverse vibrations. In (1.2), u denotes the displacement, $f(x, u)$ the external force and b the initial tension while a is related to the intrinsic properties of the string, such as Young's modulus. We have to point out that such nonlocal problems also appear in other fields as biological systems, where u describes a process which depends on the average of itself, for example, population density. For more mathematical and physical background of the problem (1.2), we refer the readers to the papers [1,2,8,9,14,13,16,18] and the references therein.

Recently, Liu and Guo in [20] considered the Kirchhoff type equation

$$-\Big(a + b\int_{\mathbb{R}^3} |\nabla u|^2\Big)\triangle u + V(x)u = f(x, u) \text{ in } \mathbb{R}^3, \tag{1.5}$$

where a, $b > 0$ are constants, $V : \mathbb{R}^3 \to \mathbb{R}$ may not be radially symmetric, and $f(x, u)$ is asymptotically linear with respect to u at infinity. They proved that problem (1.5) has a positive solution under some technical assumptions on V and f. Later, Wu and Liu in [30] studied problem (1.5) with the nonlinearity

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