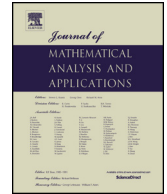




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Existence and concentrating behavior of solutions for Kirchhoff type equations with steep potential well ☆

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ABSTRACT

In this paper, we consider the following Kirchhoff type equations

$$\begin{cases} -(a + b \int_{\mathbb{R}^3} |\nabla u|^2) \Delta u + \lambda V(x)u = q(x)f(u) \text{ in } \mathbb{R}^3, \\ u \in H^1(\mathbb{R}^3), \end{cases} \quad (0.1)$$

where $a, b, \lambda > 0$, $V \in C(\mathbb{R}^3, \mathbb{R})$ is a potential well, $q(x)$ is a positive bounded function, $f(s)$ is either asymptotically linear or asymptotically 3-linear in s at infinity. Under some other suitable conditions on V, q and f , the existence, nonexistence and concentrating behavior of solutions to problem (0.1) are obtained by using variational methods. We mainly extend the results in J. Sun and T. Wu (2014) [26], which dealt with Kirchhoff type equations with positive potential well, to Kirchhoff type equations with sign-changing potential well.

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1. Introduction and main result

In this paper, we consider the following Kirchhoff type equations

$$\begin{cases} -(a + b \int_{\mathbb{R}^3} |\nabla u|^2) \Delta u + \lambda V(x)u = q(x)f(u) \text{ in } \mathbb{R}^3, \\ u \in H^1(\mathbb{R}^3) \end{cases} \quad (1.1)$$

where $a, b, \lambda > 0$, $q(x)$ is a positive bounded function, $f(s)$ is either asymptotically linear or asymptotically 3-linear in s at infinity, and the potential V satisfies the following conditions:

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- (V₁) $V \in C(\mathbb{R}^3, \mathbb{R})$ and V is bounded from below;
- (V₂) there exists $z > 0$ such that the set $\{x \in \mathbb{R}^3 : V(x) < z\}$ is nonempty and has finite measure;
- (V₃) $\Omega = \text{int}V^{-1}(0)$ is nonempty and has smooth boundary with $\bar{\Omega} = V^{-1}(0)$.

This kind of hypotheses on V was first introduced by Bartsch et al. in [3,5] to study the nonlinear Schrödinger equation. In particular, the usually concerned harmonic trapping potential $V(x) = \omega_1|x_1|^2 + \omega_2|x_2|^2 + \omega_3|x_3|^2 - \omega$ with $\omega, \omega_i > 0$ ($i = 1, 2, 3$) satisfies (V₁)–(V₃), where ω_i is called the anisotropy factor of the trap in quantum physics and trapping frequency of the i th-direction in mathematics, see [6,7,22,23].

λV represents a potential well whose depth is controlled by λ and is called the steep potential well if λ is sufficiently large. In recent years, elliptic equations with steep potential well received much attention of researchers, see [3,4,25–27,31].

Problem (1.1) arises in a famous physical context [16]. Indeed, if we set $V(x) = 0$ and replace \mathbb{R}^3 by a bounded domain $\Omega_0 \subset \mathbb{R}^3$ in (1.1), then we get the following Kirchhoff Dirichlet problem

$$\begin{cases} -\left(a + b \int_{\Omega_0} |\nabla u|^2\right) \Delta u = f(x, u), & x \in \Omega_0, \\ u = 0, & x \in \partial\Omega_0. \end{cases} \tag{1.2}$$

In recent years, a great interests have been devoted to problem (1.2), which is related to the stationary analogue of the equation

$$\rho \frac{\partial^2 u}{\partial t^2} - \left(\frac{P_0}{h} + \frac{E}{2L} \int_0^L \left|\frac{\partial u}{\partial x}\right|^2\right) \frac{\partial^2 u}{\partial x^2} = 0 \tag{1.3}$$

presented by Kirchhoff in [16] as an existence of the classical D’Alembert’s wave equations for free vibration of elastic strings. In [19], J. Lions introduced an abstract functional analysis framework to the following equation

$$u_{tt} - \left(a + b \int_{\Omega} |\nabla u|^2\right) \Delta u = f(x, u). \tag{1.4}$$

After that, (1.4) received much attention, see [1,2,8,9] and the references therein.

Kirchhoff’s model takes into account the changes in length of the string produced by transverse vibrations. In (1.2), u denotes the displacement, $f(x, u)$ the external force and b the initial tension while a is related to the intrinsic properties of the string, such as Young’s modulus. We have to point out that such nonlocal problems also appear in other fields as biological systems, where u describes a process which depends on the average of itself, for example, population density. For more mathematical and physical background of the problem (1.2), we refer the readers to the papers [1,2,8,9,14,13,16,18] and the references therein.

Recently, Liu and Guo in [20] considered the Kirchhoff type equation

$$-\left(a + b \int_{\mathbb{R}^3} |\nabla u|^2\right) \Delta u + V(x)u = f(x, u) \text{ in } \mathbb{R}^3, \tag{1.5}$$

where $a, b > 0$ are constants, $V : \mathbb{R}^3 \rightarrow \mathbb{R}$ may not be radially symmetric, and $f(x, u)$ is asymptotically linear with respect to u at infinity. They proved that problem (1.5) has a positive solution under some technical assumptions on V and f . Later, Wu and Liu in [30] studied problem (1.5) with the nonlinearity

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