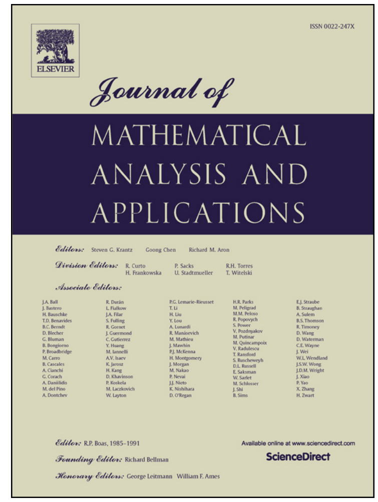

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Global regularity of the generalized liquid crystal model with fractional diffusion

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Abstract

In this paper, we consider the global regularity for a n-dimensional generalized liquid crystal system with fractional diffusion term $\zeta^2 u$ for the velocity field, where $\widehat{\zeta u}(\xi) = \frac{|\xi|^\alpha}{g(|\xi|)}\hat{u}(\xi)$ and $g^2(|\xi|) \leq C_0 \log(\mathrm{e} + |\xi|)$ for $\alpha \geq 1 + \frac{n}{2}$. This paper provides a global regularity result for the generalized liquid crystal equations, which extends the result given by Jin, Zhu and Jin to the case $\beta = 0$ for $n = 2$ (Global existence of solutions to the 2D incompressible liquid crystal flow with fractional diffusion, J. Math. Anal. Appl., 425(2015), 726-733).

Key words: liquid crystal model, fractional diffusion, global regularity.

MSC(2000): 35Q35, 35B65.

1 Introduction

In this paper, we consider the following n-dimensional incompressible generalized liquid crystal equations

$$\begin{cases} \frac{\partial u}{\partial t} + u \cdot \nabla u + \mu \wedge^{2\alpha} u + \nabla p = -\lambda \nabla \cdot (\nabla d \odot \nabla d), \\ \frac{\partial d}{\partial t} + u \cdot \nabla d + \gamma \wedge^{2\beta} d = -f(d), \\ \mathrm{div} u = 0, \\ u(x,0) = u_0,\ d(x,0) = d_0, \end{cases} \tag{1.1}$$

where u is the fluid velocity field, d stands for the macroscopic average of orientation field, p is the scalar pressure, $f(d) := (|d|^2 - 1)d$. μ is the kinematic viscosity, λ is the competition between the kinetic and potential energies, and γ is the microscopic elastic relation time for the molecular orientation field, and $\wedge$ is defined by $\widehat{\wedge f}(\xi) = |\xi|\hat{f}(\xi)$, where $\hat{f}(\xi)$ denotes the Fourier transform of $f(x)$.

Here, the following tensorial notation is used:

$$(\nabla d \odot \nabla d)_{ij} = \sum_{k=1}^{n} \partial x_i d_k \partial x_j d_k, \qquad \forall\ i,j = 1, 2 \cdots n.$$

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