J. Math. Anal. Appl. 467 (2018) 959–980

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Keller–Osserman a priori estimates and the Harnack inequality for the evolution p-Laplace equation with singular absorption term

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A R T I C L E   I N F O

Article history:
Received 31 December 2017
Available online 25 July 2018
Submitted by A. Cianchi

Keywords:
Large solutions
A priori estimates
Quasilinear parabolic equations
Singular absorption term
Harnack inequality

A B S T R A C T

In this paper we study quasilinear equations of type

$$u_t - div(|\nabla u|^{p-2}\nabla u) + V(x)f(u) = 0, \ p \geq 2, \ u \geq 0.$$

Despite the lack of comparison principle, we prove a priori estimates of Keller–Osserman type. Using these estimates we give a proof of the intrinsic Harnack inequality.

1. Introduction and main results

In this paper we consider the evolution p-Laplace equation with measurable coefficients and singular lower order term. These classes of equations have numerous applications and have been attracting attention for several decades (see, e.g. the monographs [8,9,22] and references therein).

We study nonnegative solutions to the equation

$$u_t - div A(x,t,u,\nabla u) + a_0(x,u) = 0, \tag{1.1}$$

$(x,t) \in \Omega_T = \Omega \times (0,T)$ where Ω is a domain in $\mathbb{R}^n, n \geq 3$ and $0 < T < +\infty$.

Throughout the paper we assume that the functions $A : \Omega \times \mathbb{R}^1_+ \times \mathbb{R}^1_+ \times \mathbb{R}^n \to \mathbb{R}^n$, $a_0 : \Omega \times \mathbb{R}^1_+ \to \mathbb{R}^n$ are such that $A(\cdot,\cdot,u,\varsigma)$, $a_0(\cdot,u)$ are Lebesgue measurable for all $u \in \mathbb{R}^1_+, \varsigma \in \mathbb{R}^n$, and $A(x,t,\cdot,\cdot)$, $a_0(x,\cdot)$

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https://doi.org/10.1016/j.jmaa.2018.07.046
0022-247X/

are continuous for almost all $x \in \Omega$, $t \in (0, T)$. We also assume that the following structure conditions are satisfied

$$\begin{aligned} &A(x,t,u,\varsigma)\varsigma \geq K_1|\varsigma|^p, \quad p > 2,\\ &|A(x,t,u,\varsigma)| \leq K_2|\varsigma|^{p-1},\\ &K_1 V(x) f(u) \leq a_0(x,u) \leq K_2 V(x) f(u), \end{aligned} \tag{1.2}$$

with some positive constants K_1, K_2. For the function $V(x)$ we assume that $V \in A_\infty$ Muckenhoupt class, i.e. there exist constants $q > 1$ and $c_q > 0$ such that for all $x \in \Omega$ and $B_r(x) \in \Omega$, there holds

$$V(B_r(x)) \left(\int\limits_{B_r(x)} V^{-\frac{1}{q-1}}(x)dx \right)^{q-1} \leq c_q |B_r(x)|^q, \tag{1.3}$$

where $V(B_r(x)) = \int\limits_{B_r(x)} V(y)dy, \;\; B_r(x) = \{y : |x-y| < r\}$.

We also assume that $V(x)$ belongs to the nonlinear Kato class, it means that

$$\sup_{x \in \Omega} W^V_{1,p}(x,R) < \infty, \tag{1.4}$$

where $W^V_{1,p}(x,R) = \int\limits_0^R \left(r^{p-n} \int\limits_{B_r(x)} V(y)dy \right)^{\frac{1}{p-1}} \frac{dr}{r}$.

Example 1. Function $V(x) = |x|^{-p} \ln^{-\beta_1} \frac{1}{|x|}$ satisfies conditions (1.3), (1.4) if $\beta_1 > p - 1$.

For the function f we assume that

$$f \in C^1(R^1_+), \;\; f > 0 \;\; f' \geq 0, \tag{1.5}$$

and with some $\mu > 0$ there holds

$$\frac{F_\alpha(u)}{F_\alpha(v)} \leq \left(\frac{u}{v}\right)^{1+\alpha(p-1)+\mu}, \quad 0 < u \leq v, \tag{1.6}$$

where $F_\alpha(u) = \int\limits_0^u f^\alpha(s)ds$ and $\alpha \in (0, \frac{1}{q(p+n)})$ is a fixed number.

Example 2. Function $F_\alpha(u)$ satisfies condition (1.6) with $\mu = (\beta - p + 1)\alpha > 0$, if $f(u) = u^\beta \widetilde{f}(u)$, $\beta > p - 1$ and $\widetilde{f}$ is nondecreasing continuous function. If $f(u) = u^{p-1}\widetilde{f}(u)$, where $\widetilde{f}$ satisfies condition (1.5), $\widetilde{f}(0) = 0$ and $\widetilde{f}'$ is nondecreasing, then $F_\alpha(u)$ satisfies (1.6) with $\mu = \alpha$.

In the sequel we say that a constant depends only upon the data if it depends only upon K_1, K_2, p, n, q, c_q and α.

Before formulating the main results, let us remind the reader the definition of a weak solution to equation (1.1). We say that a nonnegative function $u \in C_{loc}(0, T, L^2_{loc}(\Omega)) \bigcap L^p_{loc}(0, T, W^{1,p}_{loc}(\Omega))$ is a weak solution to equation (1.1), if for every subset $E \subset \Omega$ and every interval $[t_1, t_2] \subset (0, T]$ the following identity

$$\int\limits_E u\varphi dx \Big|_{t_1}^{t_2} + \int\limits_{t_1}^{t_2} \int\limits_E \{-u\varphi_t + A(x,t,u,\nabla u)\nabla\varphi + a_0(x,u)\varphi\}\, dxdt = 0 \tag{1.7}$$