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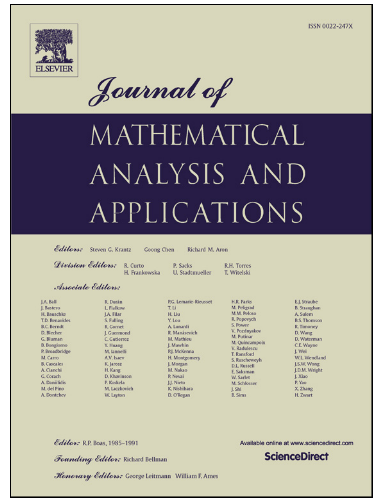

A note on strong approximation of SDEs with smooth coefficients that have at most linearly growing derivatives

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A NOTE ON STRONG APPROXIMATION OF SDEs WITH SMOOTH COEFFICIENTS THAT HAVE AT MOST LINEARLY GROWING DERIVATIVES

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Abstract. In the literature on strong approximation of SDEs, polynomial error rate results for numerical schemes are typically achieved under the assumption that the first order derivatives of the coefficients of the equation satisfy a polynomial growth condition. In the present article we show that the latter condition is far from being sufficient for polynomial error rates. We construct an SDE with smooth coefficients that have first order derivatives of at most linear growth such that the solution at the final time can at best be approximated with a logarithmic rate, whatever numerical method based on observations of the driving Brownian motion at finitely many fixed times is used. Most interestingly, it furthermore turns out that using a method that properly adjusts the number of evaluations of the driving Brownian motion to its actual path, the latter SDE can in fact be approximated with polynomial rate 1 in terms of the average number of evaluations that are used. To the best of our knowledge, this is only the second example in the literature of an SDE for which there exist adaptive methods that perform superior to non-adaptive ones with respect to the convergence rate.

1. Introduction

Let $d, m \in \mathbb{N}$, $T \in (0, \infty)$, consider a d-dimensional system of autonomous stochastic differential equations (SDE)

$$\begin{aligned} dX(t) &= \mu(X(t))\,dt + \sigma(X(t))\,dW(t), \quad t \in [0, T], \\ X(0) &= x_0 \end{aligned} \tag{1}$$

with a deterministic initial value $x_0 \in \mathbb{R}^d$, a drift coefficient $\mu\colon \mathbb{R}^d \to \mathbb{R}^d$, a diffusion coefficient $\sigma\colon \mathbb{R}^d \to \mathbb{R}^{d\times m}$ and an m-dimensional driving Brownian motion W, and assume that (1) has a unique strong solution $(X(t))_{t\in[0,T]}$. A fundamental problem in the numerical analysis of SDEs is to characterize when the solution at the final time $X(T)$ can be approximated with a polynomial error rate based on finitely many evaluations of the driving Brownian motion W in terms of explicit regularity conditions on the coefficients μ and σ.

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