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Yongjie Shi, Chengjie Yu

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CRITICAL POINTS AND SURJECTIVITY OF SMOOTH MAPS

YONGJIE SHI¹ AND CHENGJIE YU²

ABSTRACT. Let $f: M^m \to N^n$ be a smooth map between two differential manifolds with N connected, f(M) closed and $f(M) \neq N$. In this short note, we show that either all the points of M are critical points of f or the dimension the collection of all critical points of f is not less than n-1. Some consequences of this result for surjectivity of mappings are also presented.

1. INTRODUCTION

In [5], the authors obtained the following interesting result:

Theorem 1.1. Let M^m be a smooth manifold and $f: M \to \mathbb{R}^n$ be a C^1 -map with $n \ge 2$. If f has finitely many critical points and f(M) is a closed subset of \mathbb{R}^n , then f is surjective. In particular, if M is compact, then f has infinitely many critical points.

In this paper, by applying a similar trick as in the proof of Theorem 1.1 in [5] and the observation that boundary points of f(M) must be critical values, we are able to obtain a stronger conclusion:

Theorem 1.2. Let M be an m-dimensional differential manifold, N be a connected n-dimensional differential manifold, and $f : M \to N$ be a C^1 -map with f(M) closed and $f(M) \neq N$. Then either all points of M are critical points of f or the dimension of the collection of all critical points of f is not less than n - 1.

As a consequence, we have the following criterion for surjectivity of differentiable maps:

Corollary 1.1. Let M be an m-dimensional differential manifold, N be a connected n-dimensional differential manifold, and $f: M \to N$ be a C^1 -map such that

(1) f has at least one regular point;

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