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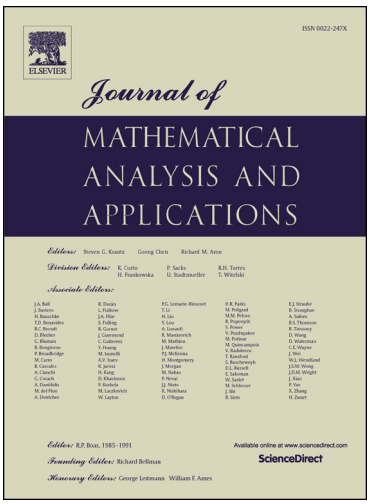

Critical points and surjectivity of smooth maps

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CRITICAL POINTS AND SURJECTIVITY OF SMOOTH MAPS

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Abstract. Let $f : M^m \to N^n$ be a smooth map between two differential manifolds with N connected, $f(M)$ closed and $f(M) \neq N$. In this short note, we show that either all the points of M are critical points of f or the dimension the collection of all critical points of f is not less than $n-1$. Some consequences of this result for surjectivity of mappings are also presented.

1. Introduction

In [5], the authors obtained the following interesting result:

Theorem 1.1. *Let M^m be a smooth manifold and $f : M \to \mathbb{R}^n$ be a C^1-map with $n \geq 2$. If f has finitely many critical points and $f(M)$ is a closed subset of $\mathbb{R}^n$, then f is surjective. In particular, if M is compact, then f has infinitely many critical points.*

In this paper, by applying a similar trick as in the proof of Theorem 1.1 in [5] and the observation that boundary points of $f(M)$ must be critical values, we are able to obtain a stronger conclusion:

Theorem 1.2. *Let M be an m-dimensional differential manifold, N be a connected n-dimensional differential manifold, and $f : M \to N$ be a C^1-map with $f(M)$ closed and $f(M) \neq N$. Then either all points of M are critical points of f or the dimension of the collection of all critical points of f is not less than $n-1$.*

As a consequence, we have the following criterion for surjectivity of differentiable maps:

Corollary 1.1. *Let M be an m-dimensional differential manifold, N be a connected n-dimensional differential manifold, and $f : M \to N$ be a C^1-map such that*

(1) *f has at least one regular point;*

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