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An improved regularity result of semi-hyperbolic patch problems for the 2-D isentropic Euler equations

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ABSTRACT

This paper investigates the regularity of a semi-hyperbolic patch problem arising from the Riemann problem for the 2-D isentropic Euler equations. We show that the solution is uniformly $C^{1,\frac{1}{6}}$ up to the sonic curve and the sonic curve is $C^{1,\frac{1}{6}}$, which improve the C^1-regularity of Song et al. [20]. We introduce a novel set of change variables which allow us to establish higher regularity results based on the ideas of characteristic decomposition and the bootstrap method.

1. Introduction

The two-dimensional isentropic compressible Euler equations reads that

$$\begin{cases} \rho_t + (\rho u)_x + (\rho v)_y = 0, \\ (\rho u)_t + (\rho u^2 + p)_x + (\rho uv)_y = 0, \\ (\rho v)_t + (\rho uv)_x + (\rho v^2 + p)_y = 0, \end{cases} \tag{1.1}$$

where ρ is the density, (u, v) is the velocity and p is the pressure given by the polytropic gas equation $p(\rho) = A\rho^\gamma$, $A > 0$ is a constant can be scaled to be one, $\gamma > 1$ is the adiabatic gas constant.

In this paper, we consider the semi-hyperbolic patch problems arising from the two-dimensional Riemann problem for (1.1). The study of two-dimensional Riemann problem to the Euler equations was initiated by Zhang and Zheng [25]. Based on the generalized characteristic analysis method and numerical experiments, they provided a set of conjectures on the configuration of solutions. However, the construction of a global solution in rigorous theory is considerably more complex due to the existence of transonic and small-scale

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structures [12,29]. The semi-hyperbolic patches are one of such kind of structures, which appear frequently in the two-dimensional Riemann problem for the Euler equations and the related models, see [5,12,29] for more details. A semi-hyperbolic patch is a region in which a family of characteristics starts on a sonic curve and ends on either a sonic curve or a transonic shock wave [21]. This type of regions may also occur in the transonic flow over an airfoil [2,3], in rarefaction wave reflection along a compressive corner [17], and in Guderley shock reflection of the von Neumann triple point paradox [22,23]. The study of the internal structure of semi-hyperbolic patches of solutions locally is quite meaningful and is an essential step for constructing global solutions of mixed-type equations in future work.

The investigation of the semi-hyperbolic patches of solutions in theoretical was started by Song and Zheng [21] for the pressure-gradient system. The semi-hyperbolic patch problems for the isentropic and isothermal Euler equations were studied, respectively, by Li and Zheng [16] and Hu et al. [7]. In [19], Song provided a different viewpoint to understand this kind of problems for the pressure-gradient system. The regularity of the semi-hyperbolic patch problems for the pressure-gradient and isentropic Euler systems were presented in [20,24]. We also refer the reader to [6,26–28] for the construction of classical sonic–supersonic solutions to the Euler system. The above works [6,7,16,19–21,24,26–28] on building smooth solutions are based on the idea of characteristic decomposition which is a powerful tool revealed in [4], see, e.g., [1,9–11,13–15,18] for more applications.

The purpose of this paper is to develop an improved regularity result of semi-hyperbolic patch problems for the two-dimensional isentropic Euler equations (1.1). We show that the global solution is uniformly $C^{1,\frac{1}{6}}$ up to the degenerate sonic boundary and the sonic curve is $C^{1,\frac{1}{6}}$-continuous. In [20], Song, Wang and Zheng verified that the semi-hyperbolic patch problem of (1.1) admits a global solution up to the sonic boundary and that the sonic boundary has C^1-regularity. We point out that the work [20] relies heavily on the known relation from [16] that $\bar{\partial}^+ c + \bar{\partial}^- c = 0$ on the sonic curve. In the current paper, we establish our regularity results without using such a relation on the sonic curve by introducing a novel set of change variables. In particular, these change variables allow us to establish higher regularity of the solution and of the sonic boundary. The approach in this paper originates from our recent work [8], where the regularity of the isothermal Euler system was studied.

The rest of the paper is organized as follows. Section 2 is devoted to delivering some preliminaries, including the variables of inclination angles and their characteristic decompositions, describing our problem and the main results in this paper. We establish the uniform boundedness of $(\bar{\partial}^+ c, \bar{\partial}^- c)$ in Section 3. In Section 4, we introduce a partial hodograph transformation and show the regularity of solutions in the partial hodograph coordinates. In the final Section 5, we complete the proof of our main theorem.

2. Preliminaries and the main results

2.1. Characteristic decompositions in angle variables

Note that system (1.1) admits self-similar solutions. In terms of self-similar variables $(\xi, \eta) = (x/t, y/t)$, (1.1) changes to

$$\begin{cases} U\rho_\xi + V\rho_\eta + \rho(u_\xi + v_\eta) = 0, \\ Uu_\xi + Vu_\eta + \left(\frac{c^2}{\gamma-1}\right)_\xi = 0, \\ Uv_\xi + Vv_\eta + \left(\frac{c^2}{\gamma-1}\right)_\eta = 0, \end{cases} \tag{2.1}$$

where $(U, V) = (u - \xi, v - \eta)$ is the pseudo-velocity and $c = \sqrt{p'(\rho)}$ is the sound speed. Assuming the flow is irrotational, i.e. $u_\eta = v_\xi$, we obtain a system in terms of (u, v)