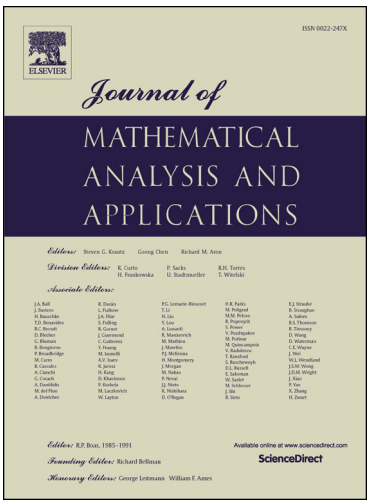
Accepted Manuscript



Global weighted Orlicz estimates for parabolic measure data problems: Application to estimates in variable exponent spaces

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PII: S0022-247X(18)30649-8
DOI: https://doi.org/10.1016/j.jmaa.2018.07.059
Reference: YJMAA 22450

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 2 June 2018

GLOBAL WEIGHTED ORLICZ ESTIMATES FOR PARABOLIC MEASURE DATA PROBLEMS: APPLICATION TO ESTIMATES IN VARIABLE EXPONENT SPACES

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Abstract. We consider a nonlinear parabolic equation with measurable nonlinearity in a nonsmooth bounded domain when the right-hand side is a finite signed Radon measure. Under optimal regularity assumptions on the nonlinearity and the boundary of the domain, we prove a global Calderón-Zygmund type estimate in weighted Orlicz spaces. As an application we obtain such an estimate in variable exponent spaces, which gives an alternative proof for this new result in the literature.

1. Introduction

There is a rich variety of physical and biological phenomena which are related to the measure data problems in the applied sciences: for instance, the flow pattern of blood in the heart [28, 31], and state-constrained optimal control problems [10, 11, 24].

In this paper we study the Cauchy-Dirichlet problem with measure data

$$\begin{cases} u_t - \operatorname{div} \mathbf{a}(Du, x, t) &= \mu \quad \text{in } \Omega_T, \\ u &= 0 \quad \text{on } \partial_p \Omega_T, \end{cases} \tag{1.1}$$

where Ω is a bounded domain of $\mathbb{R}^n$ $(n \geq 2)$, with nonsmooth boundary $\partial\Omega$, and $\Omega_T = \Omega \times (0, T]$, $T > 0$, is the space-time domain with parabolic boundary $\partial_p \Omega_T = (\partial\Omega \times [0, T]) \cup (\Omega \times \{0\})$. The nonlinearity $\mathbf{a} = \mathbf{a}(\xi, x, t) : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is assumed to be measurable in x and t and satisfies the following structure conditions:

$$\begin{cases} |\xi||D_\xi \mathbf{a}(\xi, x, t)| + |\mathbf{a}(\xi, x, t)| \leq \Lambda|\xi|, \\ \lambda|\eta|^2 \leq \langle D_\xi \mathbf{a}(\xi, x, t)\eta, \eta \rangle, \end{cases} \tag{1.2}$$

for every $x, \eta \in \mathbb{R}^n$, $t \in \mathbb{R}$, $\xi \in \mathbb{R}^n \setminus \{0\}$, and some positive constants λ, Λ. The nonhomogeneous term μ is a signed Radon measure on Ω_T with finite total variation $|\mu|(\Omega_T) < \infty$, and we assume that μ is defined in $\mathbb{R}^{n+1}$ by considering the zero extension to $\mathbb{R}^{n+1}$.

The aim of this paper is to develop a global Calderón-Zygmund type estimate for a solution to the problem (1.1) in weighted Orlicz spaces. More precisely, we deduce a weighted norm inequality for the spatial gradient of a solution to (1.1)

Date: June 2, 2018.

2010 *Mathematics Subject Classification.* Primary 35K59; Secondary 42B37, 46F30.

Key words and phrases. nonlinear parabolic equation; measure data; weighted Orlicz space; extrapolation; Calderon-Zygmund estimate; Reifenberg flat domain.

S.-S. Byun was supported by NRF-2015R1A4A1041675. J.-T. Park was supported by NRF-2017R1A2B2003877.