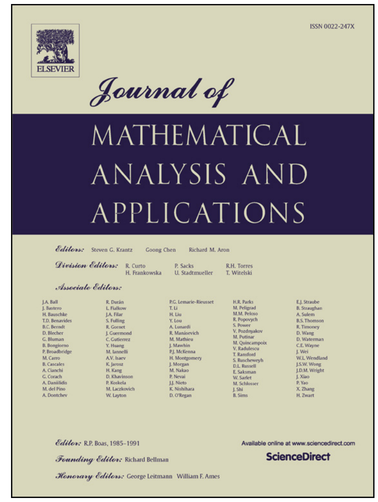

Generalised supersolutions with mass control for the Keller-Segel system with logarithmic sensitivity

Anna Zhigun

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Generalised supersolutions with mass control for the Keller-Segel system with logarithmic sensitivity

Anna Zhigun

Technische Universität Kaiserslautern, Felix-Klein-Zentrum für Mathematik
Paul-Ehrlich-Str. 31, 67663 Kaiserslautern, Germany
e-mail: zhigun@mathematik.uni-kl.de

Abstract

The existence of generalised global supersolutions with a control upon the total muss is established for the parabolic-parabolic Keller-Segel system with logarithmic sensitivity for any space dimension. It is verified that smooth supersolutions of this sort are actually classical solutions. Unlike the previously existing constructions, neither is the chemotactic sensitivity coefficient required to be small, nor is it necessary for the initial data to be radially symmetric.

Keywords: chemotaxis; generalised supersolution; global existence; logarithmic sensitivity.
MSC 2010: 35B45, 92C17, 35D30, 35D99, 35K55.

1 Introduction

Coupled reaction-diffusion-transport PDEs are a standard tool in the mathematical modelling of cell motility on the macroscale. Thereby, the diffusion-dominated systems are among the best studied analytically. Standard theory (see, e.g., [13]) ensures the existence of bounded solutions to such systems. Still, they are not always the optimal choice. Indeed, in many instances it is not the chaotic movement but, rather, the active drift of the cells towards some substance which actually dominates the motion and, as a result, may lead to a strong aggregation of the biomass density. The best known model example of such a situation is provided by the celebrated Keller-Segel system for chemotaxis [11, 12]. This parabolic-parabolic system and its parabolic-elliptic simplifications have been objects of extensive studies in recent decades. It turned out that in higher spatial dimensions the solutions to such systems can exhibit a blow-up in finite time which calls into question the global solvability. For certain parabolic-elliptic versions of the classical Keller-Segel model on the plane one was able to extend the solutions which collapse into a persistent Dirac-type singularity beyond a finite-time blow-up by constructing measure-valued continuations [16, 21]. For a detailed overview of available results concerning boundedness/blow-up, as well as other properties, of the Keller-Segel model the reader is referred to [2, 10].

Very recently a new solution concept was introduced [15] in the context of a version of the Keller-Segel system with a signal-dependent chemotactic sensitivity:

$$\begin{cases} \partial_t u = \nabla \cdot \left(\nabla u - \chi \dfrac{u}{v} \nabla v \right) & \text{in } \mathbb{R}^+ \times \Omega, & (1.1a) \\ \partial_t v = \Delta v - v + u & \text{in } \mathbb{R}^+ \times \Omega, & (1.1b) \\ \partial_\nu u = \partial_\nu v = 0 & \text{in } \mathbb{R}^+ \times \partial\Omega, & (1.1c) \\ u(0, \cdot) = u_0, \ v(0, \cdot) = v_0 & \text{in } \Omega, & (1.1d) \end{cases}$$

where Ω is a smooth bounded domain in $\mathbb{R}^n$, $n \in \mathbb{N}$, with the corresponding outer normal unit vector ν on $\partial\Omega$, and χ is a positive number. In this model prototype the cells are assumed to respond to the changes of the logarithm of the signal concentration thus following the Weber-Fechner law. Due to the saturation effect upon the chemotactic sensitivity in the presence of high levels of signal concentration, the solutions of both (1.1) and the corresponding parabolic-elliptic versions are less prone to the formation of strong singularities, such as, e.g., Dirac measures, than those of the classical Keller-Segel model. In particular, the global existence of bounded classical [2, 6–9, 14, 17, 18, 22, 26], weak [20, 22], and generalised [3, 15] solutions was established for certain ranges of parameter χ which depend upon n and, also, on whether the setting is radial-symmetric or not. On the other hand, it is known, for a parabolic-elliptic case at