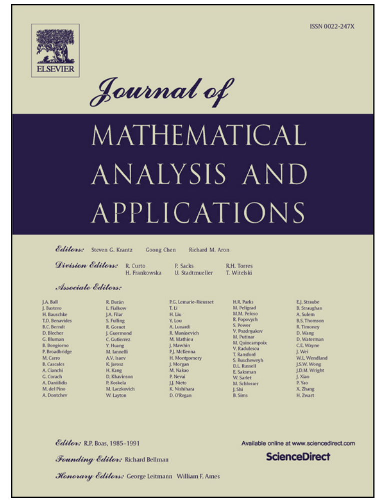

A generalized supercongruence of Kimoto and Wakayama

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A generalized supercongruence of Kimoto and Wakayama

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Abstract. In 2006, Kimoto and Wakayama discussed one kind of Apéry-like numbers which occurs in a representation of the special value of the spectral zeta function, and proposed a supercongruence conjecture on the sum of these numbers. This supercongruence conjecture was first proved by Long, Osburn and Swisher. In this paper, we extend the result of Long, Osburn and Swisher to a supercongruence modulo p^4, which was originally conjectured by Sun.

Keywords: Apéry-like number; spectral zeta function; supercongruence; Bernoulli number

MR Subject Classifications: Primary 11B65, 11A07; Secondary 05A19, 11M41

1 Introduction

For $\alpha, \beta \in \mathbb{R}$ with $\alpha\beta > 1$, the differential operator $Q_{\alpha,\beta}$ is defined as

$$Q_{\alpha,\beta} = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} \left(-\frac{1}{2}\frac{d^2}{dx^2} + \frac{1}{2}x^2 \right) + \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \left(x\frac{d}{dx} + \frac{1}{2} \right),$$

which is associated with the *non-commutative harmonic oscillator* (see [9] for more details). This differential operator defines a positive and self-adjoint operator with a discrete spectrum

$$\lambda_1 \le \lambda_2 \le \cdots \le \lambda_n \le \cdots (\to \infty).$$

Its eigenvalues are real and form a discrete set. Thus, the *spectral zeta function* can be defined as

$$\zeta_Q(s) = \sum_{n=1}^{\infty} \frac{1}{\lambda_n^s},$$

for $\Re(s) > 1$ (by analytic continuation elsewhere).

In 2006, Kimoto and Wakayama [5] introduced the Apéry-like numbers

$$J_2(n) = \sum_{k=0}^{n} (-1)^k \binom{n}{k} \binom{-\frac{1}{2}}{k}^2,$$