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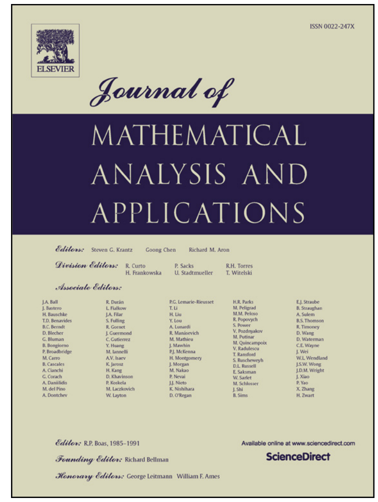

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THE ARITHMETIC DECOMPOSITION OF CENTRAL CANTOR SETS

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Abstract. Every central Cantor set of positive Lebesgue measure is the arithmetic sum of two central Cantor sets of Lebesgue measure zero. Under some mild condition this result can be strengthend by stating that the summands can be chosen to be C^s regular if the initial set is of this class.

1. Introduction

Cantor sets (by which name in this note we understand non-empty bounded nowhere dense perfect subsets of $\mathbb{R}$) and their arithmetic sums appear in many different settings. Related with the study of bifurcations of generic one-parameter families of surface diffeomorfisms having a generic homoclinic tangency at a parameter value, J. Palis [10] asked if the arithmetic sum (or difference) of two Cantor sets, both with Lebesgue measure zero is either of Lebesgue measure zero or it contains an interval. This is false in full generality (see [11], [2], [1]), but Moreira and Yoccoz in the ingenious paper [9] have shown that it is generically true for dynamically defined Cantor sets.

The main result of our note shows that every central Cantor set of positive Lebesgues measure gives rise to a counterexample to the Palis hypothesis. In our second theorem we use the powerful characterization of degree of regularity of central Cantor sets - established in [2] - for strengthening our decomposition result in terms of regularity.

Given a convergent series $\sum a_n$ of positive and nonincreasing terms, we will denote the set of its subsums by $E(a_n)$, that is,

$$E(a_n) := \left\{ x \in \mathbb{R} : \exists A \subset \mathbb{N} \qquad x = \sum_{n \in A} a_n \right\}.$$

The n-th remainder of the series $\sum a_n$ will be denoted by r_n, that is, $r_n := \sum_{k=n+1}^{\infty} a_k$. The classical result of S. Kakeya says that if $a_n > r_n$ for all n (we say in this case that $\sum a_n$ is fast convergent) then $E(a_n)$ is a Cantor set and its Lebesgue measure is $\mu E(a_n) = \lim_n 2^n r_n$ [7, 6, 8, 5].

A central Cantor set in $\mathbb{R}$ is constructed from a sequence $(\lambda_i)_{i \in \mathbb{N}}$, with $\lambda_i < \frac{1}{2}$ for all $i \in \mathbb{N}$, in the following way: choose an arbitrary closed interval K_0 and delete the middle open interval of length $|K_0| - 2\lambda_1 |K_0|$ leaving two intervals each of length $\lambda_1 |K_0|$. Call this process "process λ_1 on K_0". Let K_1 be the union of the remaining two intervals. Now do the "process λ_2 on

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