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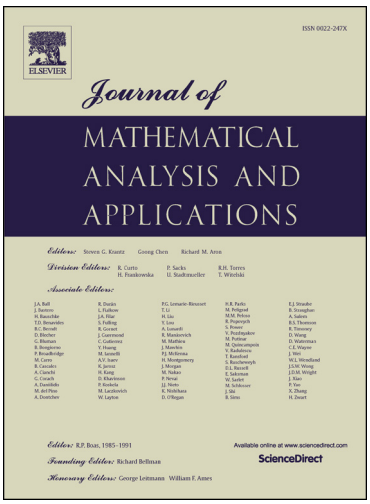

Hardy-type theorems on Fourier transforms revised

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HARDY-TYPE THEOREMS ON FOURIER TRANSFORMS REVISED

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Abstract. We obtain new conditions on periodic integrable functions so that their transformed Fourier series belong to L_p. This improves the classical Hardy and Bellman results. A counterpart for the Fourier transforms is also established. Our main tool is a new extension of the Hausdorff–Young–Paley inequality for Fourier transforms.

1. Introduction

1.1. **Transformed Fourier series.** The famous Hardy's result [12] on the transformed Fourier series reads as follows.

Theorem A. *Let the series*

$$\frac{a_0(f)}{2}+\sum_{k=1}^{\infty}(a_k(f)\cos kx+b_k(f)\sin kx) \tag{1}$$

be the Fourier series of a function $f\in L_p(\mathbb{T})$, $1<p<\infty$. Then the series

$$\frac{a_0^H(f)}{2}+\sum_{k=1}^{\infty}(a_k^H(f)\cos kx+b_k^H(f)\sin kx), \tag{2}$$

where

$$a_k^H(f)=\frac{1}{k+1}\sum_{l=0}^{k}a_l(f),\qquad b_k^H=\frac{1}{k+1}\sum_{l=1}^{k}b_l(f),\qquad k=1,2,\dots$$

is the Fourier series of a function $F\in L_p(\mathbb{T})$.

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