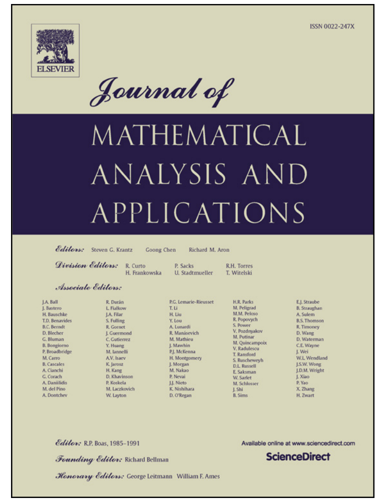
Accepted Manuscript



Energy decay of a microbeam model with a locally distributed nonlinear feedback control

Patricio Guzmán

PII: S0022-247X(18)30582-1
DOI: https://doi.org/10.1016/j.jmaa.2018.07.006
Reference: YJMAA 22397

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 29 January 2018

Energy decay of a microbeam model with a locally distributed nonlinear feedback control

Patricio Guzmán*

Abstract

In this paper we address the problem of internal stabilization of the deflection of a microbeam, which is modeled by a sixth–order hyperbolic equation. Employing multiplier techniques and an integral inequality, we prove that a locally distributed nonlinear feedback control forces the energy associated to the deflection to decay exponentially or polynomially to zero. As a consequence of this, the deflection goes to the rest position as the time goes to infinity.

2010 Mathematics Subject Classification: 35B40, 35L35, 74K10, 93B52.

Keywords: Microbeam model, hyperbolic equation, internal stabilization, locally distributed nonlinear feedback control, exponential energy decay, polynomial energy decay.

1 Introduction

A microbeam is a beam whose dimensions are in the order of a few microns. According to [26, Chapter 6], the microbeams are perhaps the most common structural component used in micro–electro–mechanical systems (MEMS) such as actuators, filters, resonators and sensors.

The deflection $z = z(t,x)$ of a clamped microbeam of density $\rho > 0$, cross–sectional area $A > 0$, Young's modulus $E > 0$, area moment of inertia $I > 0$, shear modulus $G > 0$ and length $L > 0$ being subjected to a distributed load $f = f(t,x)$ can be modeled by

$$\begin{cases} \rho A z_{tt} + M_1 z_{xxxx} - M_2 z_{xxxxxx} = f, & (t,x) \in (0,\infty) \times (0,L), \\ z(t,0) = z_x(t,0) = z_{xx}(t,0) = 0, & t \in (0,\infty), \\ z(t,L) = z_x(t,L) = z_{xx}(t,L) = 0, & t \in (0,\infty), \\ z(0,x) = z_0(x), & x \in (0,L), \\ z_t(0,x) = z_1(x), & x \in (0,L). \end{cases} \tag{1.1}$$

This model has been derived in [12, Section 3] and [17, Section 4] by using the modified strain gradient elasticity theory developed in [19, Section 2] together with Hamilton's Principle. Here

$$M_1 = EI + GA\left(2l_0^2 + \frac{8}{15}l_1^2 + l_2^2\right) \quad \text{and} \quad M_2 = GA\left(2l_0^2 + \frac{4}{5}l_1^2\right),$$

*Departamento de Ingeniería Matemática y Centro de Modelamiento Matemático, Facultad de Ciencias Físicas y Matemáticas, Universidad de Chile, Santiago, Chile. E-mail: pguzman@dim.uchile.cl