

# Accepted Manuscript

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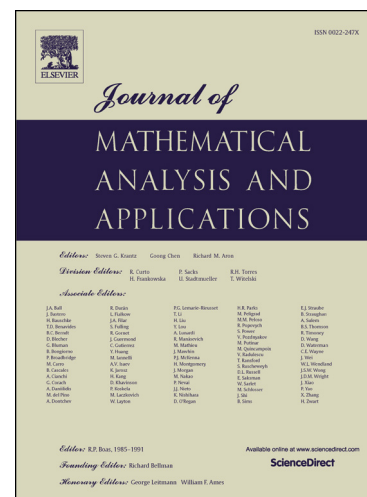
PII: S0022-247X(18)30568-7  
DOI: <https://doi.org/10.1016/j.jmaa.2018.06.067>  
Reference: YJMAA 22383

To appear in: *Journal of Mathematical Analysis and Applications*

Received date: 18 April 2018

Please cite this article in press as: F. Marceca, Some remarks on non-symmetric polarization, *J. Math. Anal. Appl.* (2018), <https://doi.org/10.1016/j.jmaa.2018.06.067>

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## SOME REMARKS ON NON-SYMMETRIC POLARIZATION

FELIPE MARCECA

ABSTRACT. Let  $P : \mathbb{C}^n \rightarrow \mathbb{C}$  be an  $m$ -homogeneous polynomial given by

$$P(x) = \sum_{1 \leq j_1 \leq \dots \leq j_m \leq n} c_{j_1 \dots j_m} x_{j_1} \dots x_{j_m}.$$

Defant and Schlütters defined a non-symmetric associated  $m$ -form  $L_P : (\mathbb{C}^n)^m \rightarrow \mathbb{C}$  by

$$L_P(x^{(1)}, \dots, x^{(m)}) = \sum_{1 \leq j_1 \leq \dots \leq j_m \leq n} c_{j_1 \dots j_m} x_{j_1}^{(1)} \dots x_{j_m}^{(m)}.$$

They estimated the norm of  $L_P$  on  $(\mathbb{C}^n, \|\cdot\|)^m$  by the norm of  $P$  on  $(\mathbb{C}^n, \|\cdot\|)$  times a  $(c \log n)^{m^2}$  factor for every 1-unconditional norm  $\|\cdot\|$  on  $\mathbb{C}^n$ . A symmetrization procedure based on a card-shuffling algorithm which (together with Defant and Schlütters' argument) brings the constant term down to  $(cm \log n)^{m-1}$  is provided. Regarding the lower bound, it is shown that the optimal constant is bigger than  $(c \log n)^{m/2}$  when  $n \gg m$ . Finally, the case of  $\ell_p$ -norms  $\|\cdot\|_p$  with  $1 \leq p < 2$  is addressed.

## 1. INTRODUCTION

Let  $P : \mathbb{C}^n \rightarrow \mathbb{C}$  be an  $m$ -homogeneous polynomial. It is well-known that there is a unique symmetric  $m$ -linear form  $B : (\mathbb{C}^n)^m \rightarrow \mathbb{C}$ , such that  $B(x, \dots, x) = P(x)$  for all  $x \in \mathbb{C}^n$ . Moreover, the *polarization formula* gives an expression for the  $m$ -linear form  $B$  in terms of  $P$  (see e.g. [3, Section 1.1]). In fact, for every  $x^{(1)}, \dots, x^{(m)} \in \mathbb{C}^n$ , we have

$$B(x^{(1)}, \dots, x^{(m)}) = \frac{1}{2^m m!} \sum_{\varepsilon \in \{-1, 1\}^m} P(\varepsilon_1 x^{(1)} + \dots + \varepsilon_m x^{(m)}).$$

It follows from this identity that

$$\sup_{\|x^{(k)}\| \leq 1} |B(x^{(1)}, \dots, x^{(m)})| \leq e^m \sup_{\|x\| \leq 1} |P(x)|, \quad (1)$$

for any norm  $\|\cdot\|$  in  $\mathbb{C}^n$ .

In [2], Defant and Schlütters defined a non-symmetric  $m$ -linear form  $L_P$  arising from a given  $m$ -homogeneous polynomial  $P$ . More precisely, for an  $m$ -homogeneous polynomial  $P : \mathbb{C}^n \rightarrow \mathbb{C}$  defined by

$$P(x) = \sum_{1 \leq j_1 \leq \dots \leq j_m \leq n} c_{j_1 \dots j_m} x_{j_1} \dots x_{j_m},$$

its associated  $m$ -linear form  $L_P : (\mathbb{C}^n)^m \rightarrow \mathbb{C}$  is given by

$$L_P(x^{(1)}, \dots, x^{(m)}) = \sum_{1 \leq j_1 \leq \dots \leq j_m \leq n} c_{j_1 \dots j_m} x_{j_1}^{(1)} \dots x_{j_m}^{(m)}.$$

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This work has been supported by CONICET-PIP 11220130100329CO, ANPCyT PICT 2015-2299, UBACyT 20020130100474BA and a CONICET doctoral fellowship.

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