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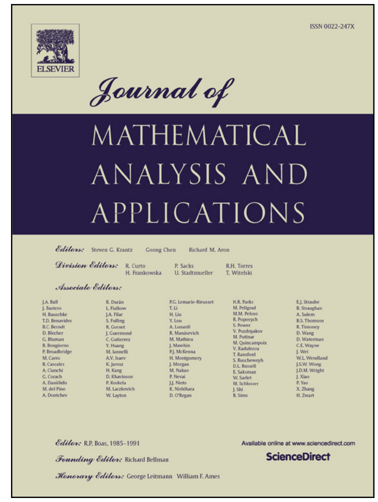

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SOME REMARKS ON NON-SYMMETRIC POLARIZATION

FELIPE MARCECA

Abstract. Let $P : \mathbb{C}^n \to \mathbb{C}$ be an m-homogeneous polynomial given by

$$P(x) = \sum_{1 \le j_1 \le \ldots \le j_m \le n} c_{j_1 \ldots j_m} x_{j_1} \ldots x_{j_m}.$$

Defant and Schlüters defined a non-symmetric associated m-form $L_P : (\mathbb{C}^n)^m \to \mathbb{C}$ by

$$L_P\left(x^{(1)}, \ldots, x^{(m)}\right) = \sum_{1 \le j_1 \le \ldots \le j_m \le n} c_{j_1 \ldots j_m} x^{(1)}_{j_1} \ldots x^{(m)}_{j_m}.$$

They estimated the norm of L_P on $(\mathbb{C}^n, \|\cdot\|)^m$ by the norm of P on $(\mathbb{C}^n, \|\cdot\|)$ times a $(c \log n)^{m^2}$ factor for every 1-unconditional norm $\|\cdot\|$ on $\mathbb{C}^n$. A symmetrization procedure based on a card-shuffling algorithm which (together with Defant and Schlüters' argument) brings the constant term down to $(cm \log n)^{m-1}$ is provided. Regarding the lower bound, it is shown that the optimal constant is bigger than $(c \log n)^{m/2}$ when $n \gg m$. Finally, the case of ℓ_p-norms $\|\cdot\|_p$ with $1 \le p < 2$ is addressed.

1. Introduction

Let $P : \mathbb{C}^n \to \mathbb{C}$ be an m-homogeneous polynomial. It is well-known that there is a unique symmetric m-linear form $B : (\mathbb{C}^n)^m \to \mathbb{C}$, such that $B(x, \ldots, x) = P(x)$ for all $x \in \mathbb{C}$. Moreover, the *polarization formula* gives an expression for the m-linear form B in terms of P (see e.g. [3, Section 1.1]). In fact, for every $x^{(1)}, \ldots, x^{(m)} \in \mathbb{C}$, we have

$$B\left(x^{(1)}, \ldots, x^{(m)}\right) = \frac{1}{2^m m!} \sum_{\varepsilon \in \{-1,1\}^m} P\left(\varepsilon_1 x^{(1)} + \ldots + \varepsilon_m x^{(m)}\right).$$

It follows from this identity that

$$\sup_{\|x^{(k)}\| \le 1} \left| B\left(x^{(1)}, \ldots, x^{(m)}\right) \right| \le e^m \sup_{\|x\| \le 1} |P(x)|, \tag{1}$$

for any norm $\|\cdot\|$ in $\mathbb{C}^n$.

In [2], Defant and Schlüters defined a non-symmetric m-linear form L_P arising from a given m-homogeneous polynomial P. More precisely, for an m-homogeneous polynomial $P : \mathbb{C}^n \to \mathbb{C}$ defined by

$$P(x) = \sum_{1 \le j_1 \le \ldots \le j_m \le n} c_{j_1 \ldots j_m} x_{j_1} \ldots x_{j_m},$$

its associated m-linear form $L_P : (\mathbb{C}^n)^m \to \mathbb{C}$ is given by

$$L_P\left(x^{(1)}, \ldots, x^{(m)}\right) = \sum_{1 \le j_1 \le \ldots \le j_m \le n} c_{j_1 \ldots j_m} x^{(1)}_{j_1} \ldots x^{(m)}_{j_m}.$$

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