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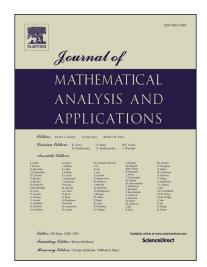
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ACCEPTED MANUSCRIPT

On a generalisation of Krein's example

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We generalise a classical example given by Krein in 1953. We compute the difference of the resolvents and the difference of the spectral projections explicitly. We further give a full description of the unitary invariants, i.e., of the spectrum and the multiplicity. Moreover, we observe a link between the difference of the spectral projections and Hankel operators.

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1 Introduction and main results

1.1 Introduction

The spectral shift function was introduced at a formal level by Lifshits [16]; Krein presented in [14] a rigorous definition of the spectral shift function $\xi = \xi(\bullet, A_1, A_0) \in \mathsf{L}_1(\mathbb{R})$ defined via

$$\operatorname{tr}(\chi(A_1) - \chi(A_0)) = \int_{\mathbb{R}} \chi'(\vartheta) \xi(\vartheta) d\vartheta,$$

whenever χ belongs to a suitable class of functions and $A_1 - A_0$ is of trace class. In a naive definition, one would choose the characteristic function $\chi = \mathbb{1}_{(-\infty,\vartheta)}$, as the above formula then becomes formally

$$\operatorname{tr}\left(\mathbb{1}_{(-\infty,\vartheta)}(A_1) - \mathbb{1}_{(-\infty,\vartheta)}(A_0)\right) = \xi(\vartheta). \tag{1.1}$$

Unfortunately,¹ formula (1.1) is not true: even if $A_1 - A_0$ is a rank 1 perturbation (and hence of trace class), the difference of the spectral projections $\mathbb{1}_{(-\infty,\vartheta)}(A_1) - \mathbb{1}_{(-\infty,\vartheta)}(A_0)$ need not to be of trace class, i.e., the left hand side of (1.1) is not defined. Krein presented such an example in his paper [14], where $A_1 = (H+1)^{-1}$ and $A_0 = (H^D+1)^{-1}$ are the resolvents at the spectral point -1 of the Neumann and Dirichlet Laplacian $H = \left(-\frac{d^2}{dt^2}\right)^N$ and $H^D = \left(-\frac{d^2}{dt^2}\right)^D$

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¹Maybe this is a fortune as it gave rise to new research . . .

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