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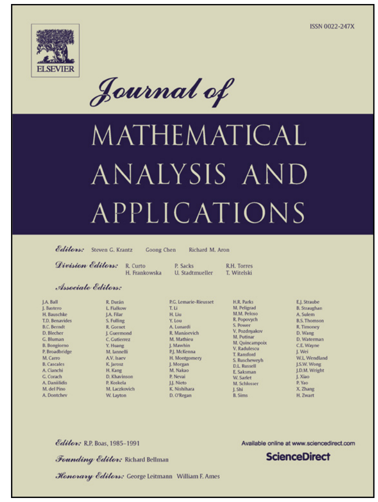

On a generalisation of Krein's example

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On a generalisation of Krein's example

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We generalise a classical example given by Krein in 1953. We compute the difference of the resolvents and the difference of the spectral projections explicitly. We further give a full description of the unitary invariants, i. e., of the spectrum and the multiplicity. Moreover, we observe a link between the difference of the spectral projections and Hankel operators.

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1 Introduction and main results

1.1 Introduction

The spectral shift function was introduced at a formal level by Lifshits [16]; Krein presented in [14] a rigorous definition of the spectral shift function $\xi = \xi(\bullet, A_1, A_0) \in \mathsf{L}_1(\mathbb{R})$ defined via

$$\operatorname{tr}(\chi(A_1) - \chi(A_0)) = \int_{\mathbb{R}} \chi'(\vartheta)\xi(\vartheta)\,\mathrm{d}\vartheta,$$

whenever χ belongs to a suitable class of functions and $A_1 - A_0$ is of trace class. In a naive definition, one would choose the characteristic function $\chi = \mathbb{1}_{(-\infty,\vartheta)}$, as the above formula then becomes formally

$$\operatorname{tr}\bigl(\mathbb{1}_{(-\infty,\vartheta)}(A_1) - \mathbb{1}_{(-\infty,\vartheta)}(A_0)\bigr) = \xi(\vartheta). \tag{1.1}$$

Unfortunately,[1] formula (1.1) is not true: even if $A_1 - A_0$ is a rank 1 perturbation (and hence of trace class), the difference of the spectral projections $\mathbb{1}_{(-\infty,\vartheta)}(A_1) - \mathbb{1}_{(-\infty,\vartheta)}(A_0)$ need not to be of trace class, i. e., the left hand side of (1.1) is not defined. Krein presented such an example in his paper [14], where $A_1 = (H+1)^{-1}$ and $A_0 = (H^{\mathrm{D}}+1)^{-1}$ are the resolvents at the spectral point -1 of the Neumann and Dirichlet Laplacian $H = \bigl(-\frac{\mathrm{d}^2}{\mathrm{d}t^2}\bigr)^{\mathrm{N}}$ and $H^{\mathrm{D}} = \bigl(-\frac{\mathrm{d}^2}{\mathrm{d}t^2}\bigr)^{\mathrm{D}}$

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[1]Maybe this is a fortune as it gave rise to new research ...