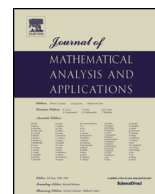




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Magnetic curves in tangent sphere bundles II

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ABSTRACT

We study contact magnetic curves in the unit tangent sphere bundle over the Euclidean plane. In particular, we obtain all contact magnetic curves which are slant.

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1. Introduction and preliminaries

As is well known, unit tangent sphere bundle over Riemannian manifolds admits the so-called standard contact metric structure. In our previous paper [12] we have developed a general theory of magnetic curves in unit tangent sphere bundles. In addition we studied magnetic curves in the unit tangent bundle US^2 of the unit 2-sphere S^2 . As a continuation of [12], in this paper, we study magnetic curves in the unit tangent sphere bundle UE^2 of the Euclidean plane E^2 . In particular, we obtain all contact normal magnetic curves on UE^2 , which satisfy the conservation law. Because the unit tangent sphere bundle UE^2 may be identified as a contact metric manifold with the motion group $E(2)$ of the Euclidean plane E^2 , we do some investigations in $E(2)$.

1.1. Magnetic curves

Magnetic curves represent, in physics, the trajectories of charged particles moving on a Riemannian manifold under the action of the magnetic fields. Let (M, g) be a Riemannian manifold and let F be a

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closed 2-form on M (often called a *magnetic field* on M). A *magnetic curve* represents a solution of a second order differential equation

$$\nabla_{\gamma'} \gamma' = \phi \gamma', \quad (1.1)$$

where ∇ denotes the Levi-Civita connection on M and ϕ is a skew-symmetric $(1,1)$ tensor field associated to F , that is $F(X, Y) = g(\phi X, Y)$ for any vector fields X, Y on M . See e.g. [1]. Such curves are sometimes called also *magnetic geodesics* since the Lorentz equation generalizes the equation of geodesics under arc-length parametrization, namely, $\nabla_{\gamma'} \gamma' = 0$. The equation (1.1) is usually known as the *Lorentz equation*. However, in contrast to the geodesics, magnetic curves cannot be rescaled, because the trajectory of a charged particle depends on the speed $|\gamma'|$. Nevertheless, magnetic curves have constant speed, and hence constant energy, since $\frac{d}{ds} g(\gamma', \gamma') = 2g(\phi \gamma', \gamma') = 0$.

And now, as usual, we restrict our investigation to a single energy level and we consider only unit speed magnetic curves together with a *strength* $q \in \mathbb{R}$. Therefore, from now on, we study *normal magnetic curve* (i.e. unit speed) satisfying the Lorentz equation

$$\nabla_{\dot{\gamma}} \dot{\gamma} = q \phi \dot{\gamma}, \quad (1.2)$$

where by **dot** we denote the derivative with respect to the arc-length parameter s .

1.2. Almost contact metric structures

A (φ, ξ, η) -structure on a manifold M is defined by a field φ of endomorphisms of tangent spaces, a vector field ξ and a 1-form η satisfying

$$\eta(\xi) = 1, \quad \varphi^2 = -I + \eta \otimes \xi, \quad \varphi \xi = 0, \quad \eta \circ \varphi = 0.$$

If (M, φ, ξ, η) admits a compatible Riemannian metric g , namely

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y),$$

for all $X, Y \in \Gamma(TM)$, then M is said to have an *almost contact metric structure*, and $(M, \varphi, \xi, \eta, g)$ is called an *almost contact metric manifold*. Consequently, we have that ξ is unitary and η is metrically dual to ξ , i.e., $\eta(X) = g(\xi, X)$, for any $X \in \Gamma(TM)$. The vector field ξ is often called the Reeb vector field, even though, this name is used for the contact case.

We define a 2-form Ω on $(M, \varphi, \xi, \eta, g)$ by

$$\Omega(X, Y) = g(X, \varphi Y),$$

for all $X, Y \in \Gamma(TM)$, called the *fundamental 2-form* of the almost contact metric structure (φ, ξ, η, g) .

The fundamental 2-form is not always closed. However, there are several classes of almost contact metric manifolds with closed fundamental 2-form. For more details see e.g. [3].

Let us remember some more definitions: An almost contact metric manifold is said to be:

- (1) A *contact metric manifold* if $\Omega = d\eta$.
- (2) An α -*Sasakian manifold* if there exists a constant α such that it satisfies

$$(\nabla_X \varphi)Y = \alpha\{g(X, Y)\xi - \eta(Y)X\}$$

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