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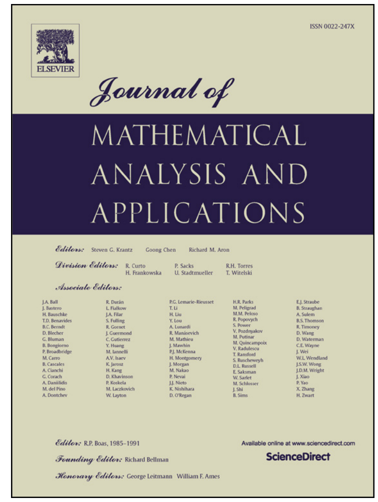

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General stability of a viscoelastic variable coefficients plate equation with time-varying delay in localized nonlinear internal feedback

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Abstract. This paper is concerned with the decay properties of a memory-type Euler-Bernoulli plate equation with variable coefficients and time-varying delay in localized nonlinear internal feedback. By using piecewise multiplier method and Riemannian geometric analysis, we establish the explicit and general decay rates of system whose nonlinear internal feedbacks act on a suitable subregion.

Keywords: Thermoelastic; nonlinear internal feedbacks; time-varying delay; Riemannian manifold

1 Introduction

Let $\Omega \subset \mathbb{R}^n$ $(n \geq 2)$ be an open, bounded domain with smooth boundary Γ, $\mathcal{A}$ be the operator on $C_0^\infty(\mathbb{R}^n)$ defined by

$$\mathcal{A}w = \mathrm{div}\ (A(x)\nabla w)\,,\ w \in C_0^\infty(\mathbb{R}^n),\ x \in \mathbb{R}^n,$$

where $\mathrm{div}\, X$ denotes the divergence of the vector field X in the Euclidean metric, ∇w denotes the gradient of w in the Euclidean metric, and $A(x) = (a_{ij}(x))$ denote symmetric and positive definite matrices with smooth elements $a_{ij}(x),\ i, j = 1, 2, \ldots, n$. We consider the following variable coefficients plate system with a time-varying delay in localized nonlinear internal feedbacks

$$\begin{cases} u_{tt} + \mathcal{A}^2 u - \int_0^t \sigma(t-s)\mathcal{A}^2 u(s)\mathrm{d}s & \\ \qquad +\chi_\omega(x)\left(\mu_1 a(u_t) + \mu_2 b(u_t(t-\tau(t)))\right) = 0 & \text{in } \ \Omega \times (0,\infty), \\ u = \frac{\partial u}{\partial \nu_A} = 0 & \text{on } \ \Gamma \times (0,\infty), \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x) & \text{in } \ \Omega, \\ u_t(x,t) = k_0(x,t) & \text{in } \ \Omega \times (-\tau(0), 0), \end{cases} \tag{1.1}$$

in which the initial data $(u_0,\, u_1,\, k_0)$ is in suitable space, $\mu_1 > 0$, $\mu_2 \in \mathbb{R}$ are constants, and $\chi_\omega(\cdot)$ is the characteristic function of the set $\omega \subset \Omega$. The functions $a,\ b : \bar{\Omega} \to \mathbb{R}_+$ and $\sigma : \mathbb{R}_+ \to \mathbb{R}_+$ will be specified later. The exterior normal derivative $\frac{\partial u}{\partial \nu_A}$ is denoted by

$$\frac{\partial u}{\partial \nu_A} = \langle A(x)\nabla u, \nu\rangle,$$

where $\langle \cdot, \cdot \rangle$ is the standard inner product in the Euclidean space and ν denotes the unit normal of Γ pointing toward the exterior of Ω.

There are three feedbacks making different contribution to system (1.1), the viscoelastic term, the frictional damping term and the delay term. Among these feedbacks, the viscoelastic damping arises in the theory of viscoelasticity. We recall that the viscoelastic materials are those with properties of both elasticity and viscosity. Due to the special properties, these kinds of materials can keep memory of their entire history and exhibit natural damping. That's why they are called memory-type materials (see [1]). The memory effects can be modeled mathematically by the convolution term reflecting that

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