



Periodic solutions of semilinear Duffing equations with impulsive effects



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ARTICLE INFO

Article history:

Received 7 January 2018
Available online 5 July 2018
Submitted by Y. Huang

Keywords:

Impulsive differential equations
Poincaré–Birkhoff twist theorem
Periodic solutions

ABSTRACT

In this paper we are concerned with the existence of periodic solutions for semilinear Duffing equations with impulsive effects. Firstly for its autonomous equation, any motion of the solution is same as the motion of the corresponding equation without impulses until it meets the first impulse time. Under the influence of impulses, these two motions are likely to be quite different. We introduce some reasonable assumptions on the impulsive functions to control these differences such that the information valid for the equation without impulses can always be used for the impulsive one. Basing on Poincaré–Birkhoff twist theorem, we prove the existence of infinitely many periodic solutions. Secondly, as for the nonautonomous equation where the autonomous case is taken as an auxiliary one, we find the relation between the solutions of these two equations and then obtain the existence of infinitely many periodic solutions also by Poincaré–Birkhoff twist theorem. Lastly, an example with special impulses satisfying the above assumptions is given.

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1. Introduction

We are concerned in this paper with the existence of periodic solutions for the second order impulsive differential equation

$$\begin{cases} x'' + g(x) = p(t), & t \neq t_j; \\ x(t_j+) = I(x(t_j-), x'(t_j-)), \\ x'(t_j+) = J(x(t_j-), x'(t_j-)), \end{cases} \quad j \in \mathbb{Z}, \quad (1.1)$$

where $0 \leq t_1 < 2\pi$, $g(x)$, $p(t) \in C(\mathbb{R}, \mathbb{R})$ and $p(t)$ is 2π -periodic, $I, J : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ are continuous maps, and the impulsive time is 2π -periodic, that is, $t_{j+1} = t_j + 2\pi$ for $j \in \mathbb{Z}$. We also use $x(t_j), x'(t_j)$ to denote $x'(t_j-), x(t_j-)$ for simplicity.

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¹ Partially supported by the NSFC (11571041) and the Fundamental Research Funds for the Central Universities.

This problem comes from Duffing equation

$$x'' + g(x) = p(t) \quad (1.2)$$

and there is a wide literature dealing with the existence of periodic solutions for the above Duffing equation, not only because of its physical significance, but also the application of various mathematical techniques on it, such as Poincaré–Birkhoff twist theorem in [9], [11], the variational method in [1], [16] and topological degree or index theories in [3], [4].

Under different assumptions on the function g , for example being superlinear, sublinear, semilinear and so on, there are many interesting results on the existence and multiplicity of periodic solutions of (1.2), see [8], [20], [23], [24] and the references therein. Among these, the existence problem of periodic solutions for semilinear Duffing equations challenges more attentions for its special resonance phenomenon. At resonance, equation (1.2) may have no bounded solutions, therefore the crucial point of solving this problem is to exclude the resonance, and there are also many studies on it, see [5], [6] and [14].

Recently, as impulsive equations widely arise in applied mathematics, they attract a lot of attentions and many authors study the basic theory in [2], [13], along with the existence of periodic solutions of impulsive differential equations via fixed point theory in [17], topological degree theory in [10], [22], and the variational method in [18], [25].

As we all know, the existence of impulses, even the simplest impulsive function, may cause complicated dynamic phenomena and bring great difficulties to study. The changes between the behaviors of solutions with and without impulsive effects may be great.

Here we take the simplest linear equation as an example and consider

$$x'' + x = 0 \quad (1.3)$$

with the impulsive conditions

$$x(t_j+) = 2x(t_j-), \quad x'(t_j+) = \frac{1}{2}x'(t_j-), \quad (1.4)$$

where $t_j = j\pi$ for $j \in \mathbb{Z}$. It is easy to see that without impulses, all solutions of (1.3) are 2π -periodic and satisfy

$$x^2(t) + x'^2(t) = C, \quad (1.5)$$

where C is a constant related to the initial values. However, under the influence of impulsive functions in (1.4), all solutions except for the trivial one are unbounded. In fact, with the initial point $(x(0), x'(0)) = (x_0, 0)$ ($x_0 \neq 0$), the solution at each $t_j \pm$ is located on the x -axis and the radius of the trajectory (1.5) at t_j+ becomes two times larger than the previous one at t_j- , that is, $(x(\pi+), x'(\pi+)) = (-2x_0, 0)$, $(x(2\pi+), x'(2\pi+)) = (4x_0, 0), \dots$, which implies the solution tends to infinity as $t \rightarrow +\infty$.

From the above example, we find that the existence of periodic solutions for impulsive semilinear Duffing equations deserves exploring. What impulsive functions do guarantee the existence of periodic solutions is the key point. Different from the extensive study for second order differential equations without impulsive terms, there are only a few results on the existence and multiplicity of periodic solutions for impulsive ones.

In [21], Qian et al. considered the superlinear impulsive differential equation

$$\begin{cases} x'' + g(x) = p(t, x, x'), & t \neq t_j; \\ \Delta x|_{t=t_j} = I_j(x(t_j-), x'(t_j-)), \\ \Delta x'|_{t=t_j} = J_j(x(t_j-), x'(t_j-)), & j \in \mathbb{Z}, \end{cases} \quad (1.6)$$

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