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An optimal linear approximation for a class of nonlinear operators between uniform algebras

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ABSTRACT

Assume that A, B are uniform algebras on compact Hausdorff spaces X and Y, respectively. Let $T : A \to B$ be a map (nonlinear in general) satisfying $T(A^{-1}) = B^{-1}$ and $T1 = 1$. We show that, if there exist constants $\alpha, \beta \geq 1$ such that $\beta^{-1}\|f \cdot g^{-1}\| \leq \|Tf \cdot (Tg)^{-1}\| \leq \alpha\|f \cdot g^{-1}\|$ for all $f \in A$ and $g \in A^{-1}$, then there exists a homeomorphism $\tau : \partial B \to \partial A$ between the Šilov boundaries of A and B such that $(\alpha\beta)^{-1}|f(\tau(y))| \leq |(Tf)(y)| \leq \alpha\beta|f(\tau(y))|$ for all $f \in A$ and for all $y \in \partial B$. In particular ∂A and ∂B are homeomorphic. Moreover we give an example which shows that the multiple $\alpha\beta$ in the above inequality is best possible.

1. Introduction

Let X, Y be compact Hausdorff spaces. We denote by $C(X), C(Y)$ the Banach spaces of all complex continuous functions on X, Y endowed with its usual sup-norm. The following result is well-known as the Banach–Stone theorem (see [3,27]).

Theorem 1.1. *Let T be a surjective linear isometry from the space $C(X)$ onto the space $C(Y)$. Then there is a homeomorphism $\tau : Y \to X$ and a continuous map $h \in C(Y)$ with $|h(y)| = 1$ for all $y \in Y$, such that T can be written as a weighted composition map, that is,*

$$(Tf)(y) = h(y)f(\tau(y)) \tag{1.1}$$

for all $y \in Y$ and all $f \in C(X)$.

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This classic theorem has been generalized in several directions (for more details see [4,14]), for instance, as characterizations of mappings between algebras of continuous functions to be (weighted) composition operators. Recall that given two algebras of continuous functions, $A \subset C(X)$ and $B \subset C(Y)$, and a continuous map $\tau : Y \to X$, a map $T : A \to B$ is said to be: a *composition operator* on Y if $(Tf)(y) = f(\tau(y))$, a *composition operator in modulus* on Y if $|(Tf)(y)| = |f(\tau(y))|$, and a *weighted composition operator* on Y if there exists a non-vanishing function $h \in C(Y)$ so that $(Tf)(y) = h(y)f(\tau(y))$.

Some results on approximative composition operators have been obtained. The first important result in this direction was obtained by Amir [1] and Cambern [7,8]. They considered a linear isomorphism $T : C_0(X) \to C_0(Y)$ for compact Hausdorff X, Y in [1] and locally compact Hausdorff in [7,8] satisfying the approximate condition $\|T\| \cdot \|T^{-1}\| < 2$, instead of being an isometry, and proved that the underlying spaces are homeomorphic, and furthermore the universal constant 2 is optimal (see [9,11]). Moreover, Cidral, Galego and Rincán-Villamizar [10] extended the theorem of Amir and Cambern to the vector-valued function spaces. In 1989, Jarosz [19] obtained a nonlinear version of the theorem of Amir and Cambern (see also [12,15]) which says that any bi-Lipschitz map T from $C_0(X)$ onto $C_0(Y)$ with bi-Lipschitz constant less than an absolute constant ϵ_0 is a perturbation of a composition operator and moreover X and Y are homeomorphic. In 2005, Dutrieux and Kalton [12] proved that the absolute constant ϵ_0 of Jarosz's result may be taken to be 17/16. In 2011, the constant was improved to 6/5 by Gorak [15]. Recently, Galego and Porto da Silva [13] gave an optimal estimate of the constant. They showed that a bijective coarse (M, L)-quasi-isometry from $C(X)$ onto $C(Y)$ with $M < \sqrt{2}$ can be approximated, in some sense, by a weighted composition operator on Y.

On the other hand, this subject has been studied in the "multiplicative" direction by many mathematicians (see, e.g., [16,21,22,24,25,28]). Most of these results were established in the context of uniform algebras. A closed subalgebras A of $C(X)$ is called a *uniform algebra* on X, if A separates points of X (for any pair of distinct points $x_1, x_2 \in X$ there exists $f \in A$ such that $f(x_1) \neq f(x_2)$) and contains the constant functions. In 2005, Rao and Roy [24] showed that a surjective map $T : A \to A$, where A is a uniform algebra on the maximal ideal space, is a weighted composition operator if it is *spectrally-multiplicative*, that is $\sigma((Tf)(Tg)) = \sigma(fg)$ for all $f, g \in A$, which extended a result of Molnár [22] where $A = C(X)$. After that, Lambert, Luttman and Tonev [20] proved the following theorem. Before stating the theorem, we need to introduce a few notions. For $f \in C(X)$, we define $M(f) = \{z \in X : |f(z)| = \|f\|\}$. A function $f \in C(X)$ is a *peaking function*, if $f(x) = 1$ for all $x \in M(f)$. Let A be a subset of $C(X)$ and $\mathcal{F}(A)$ denotes the set of all peaking functions in A.

Theorem 1.2. *Let A and B be uniform algebras. If a map $T : A \to B$ preserves the peaking functions (i.e., $T(\mathcal{F}(A)) = \mathcal{F}(B)$) and satisfies*

$$\|Tf \cdot Tg\| = \|fg\| \tag{1.2}$$

for all $f, g \in A$, then there exists a homeomorphism $\tau : \delta B \to \delta A$ between the Choquet boundaries (= the strong boundaries) of A and B such that

$$|(Tf)(y)| = |f(\tau(y))| \tag{1.3}$$

for all $y \in \delta B$ and all $f \in A$.

A subset $\Omega \subset X$ is said to be a *boundary* for A if for every $f \in A$ there exists $x \in \Omega$ such that $|f(x)| = \|f\|$. If there exists a unique minimal closed boundary for A, it is called the *Šilov boundary* for A which is denoted by ∂A. A point $x_0 \in X$ is called *weak boundary point* of A if for each neighborhood U of x_0, there exists $f \in A$ such that $M(f) \subset U$. We denote by σA the set of all weak boundary points of A. A point $x_0 \in X$ is called *strong boundary point* of A if for each neighborhood U of x_0, there exists $f \in A$

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