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The same class of stationary solutions to some multidimensional kinetic systems with extensive background density

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ABSTRACT

In this study, we investigate the existence and uniqueness of the same class of stationary solutions for some kinetic systems comprising the Vlasov–Poisson–Fokker–Planck system, Vlasov–Poisson–Boltzmann system, and Vlasov–Maxwell–Boltzmann system in arbitrary space dimensions $N(N \geq 3)$. Essentially, the problem can be reduced to solving the problem of a second order elliptic equation with exponential nonlinearity. This result had been proved in three spatial dimensions but the extension to a higher-dimensional setting makes the existence proof nontrivial. In particular, it necessary to highlight that the requirement regarding the background density function is relaxed compared with previous studies in the case of three spatial dimensions.

Contents

1. Introduction

In this study, we investigate the existence and uniqueness of the same class of stationary solutions for some kinetic systems comprising the Vlasov–Poisson–Fokker–Planck (VPFP) system, Vlasov–Poisson–Boltzmann (VPB) system, and Vlasov–Maxwell–Boltzmann (VMB) system in arbitrary space dimensions $N(N \geq 3)$. Many studies have investigated this problem for the three-dimensional case, but the requirement regarding the background density function is relaxed compared with these previous investigations. The extension is

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nontrivial and it can be achieved successfully by seeking appropriate functions $h(\sigma)$ and $a(\sigma)$, as stated in Assumptions $(\mathcal{A}_1)$–$(\mathcal{A}_3)$ in the following. Essentially, the problem can be reduced to solving the following problem for a second order elliptic equation with exponential nonlinearity:

$$\Delta u + \mathrm{e}^{-u} = \rho(x), \qquad x \in \mathbb{R}^N, \tag{1.1}$$

$$u(x) \to 0, \qquad \text{as } |x| \to \infty. \tag{1.2}$$

In fact, Equation (1.1) can be recovered from the VPFP system, VPB system, and VMB system, and the solution u to the (1.1) and (1.2) can be used to construct the stationary solution to the three systems mentioned above. The rigorous and detailed derivation in the case of three dimensions was given by [11,25,27,28] and it can be summarized in the following manner.

Fact 1. *Equation (1.1) is recovered from the three-dimensional VPFP system (see [28]).*

The VPFP system with all physical coefficients taken as 1 is written as:

$$\partial_t F + v \cdot \nabla_x F + \nabla_x \Phi \cdot \nabla_v F = \nabla_v \cdot (vF) + \Delta_v F, \tag{1.3}$$

$$\Delta_x \Phi = \int\limits_{\mathbb{R}^3} F dv - \rho(x). \tag{1.4}$$

From [6,16,17,28], the stationary solutions to the VPFP system (1.3) and (1.4) are given by:

$$F = \alpha(x) \cdot \mu(v), \tag{1.5}$$

where $\alpha(x)$ is determined in the following and

$$\mu(v) = \frac{1}{(2\pi)^{\frac{3}{2}}} \exp\left(-\frac{1}{2}|v|^2\right). \tag{1.6}$$

By substituting (1.5) into (1.3), we have

$$(v \cdot \nabla_x \alpha)\mu - \alpha \nabla_x \Phi \cdot v\mu = 0,$$

which will be satisfied if and only if:

$$\nabla_x \alpha = \alpha \nabla_x \Phi. \tag{1.7}$$

F is a density so the condition that $\alpha(x) \geq 0$ must be satisfied. For the set where $\alpha(x) > 0$, we can write $\alpha = \mathrm{e}^{-u}$ and recast (1.7) in the form of:

$$\nabla_x u = -\nabla_x \Phi, \tag{1.8}$$

which together with (1.4) means that:

$$\Delta_x u = -\Delta_x \Phi = -\alpha(x) \int\limits_{\mathbb{R}^3} \mu(v) dv + \rho(x) = -\alpha(x) + \rho(x). \tag{1.9}$$

By entering $\alpha = \mathrm{e}^{-u}$ into (1.9), we obtain Equation (1.1) exactly.